

Electric Circuits

Start from the original water-lifting analogy: a battery supplies the same energy per unit charge, while the circuit determines the charge flow rate. Preserve the sequence from I, V and R through series and parallel circuits, power, meters, real batteries, Kirchhoff rules and capacitors.

TEACHING ROUTE

The explanations follow the original lesson sequence. Connections, analogies and worked reasoning remain together. Syllabus additions appear beside the ideas they support.

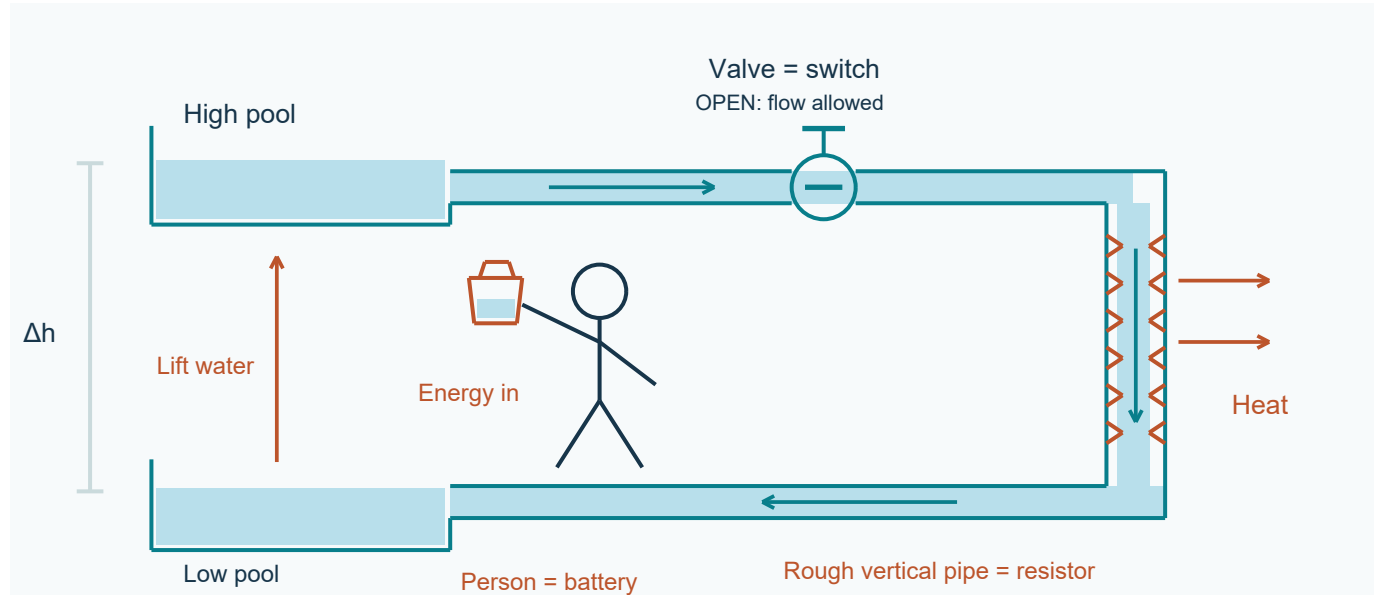
Lesson group	Focus
The physical picture	Water-lifting analogy, emf, current and resistance
Connections and power	Series and parallel ratios, bulb comparisons and meters
Real sources and loops	Internal resistance, battery graphs and current sign choices
Capacitors	Combinations, charging, discharging and the time constant
Practice and summary	Original bridge and graph reasoning, experiments and review

Predict first, draw the situation, then calculate. Use the class questions for discussion and the chapter review for independent practice. Unless stated otherwise, numerical examples use idealized models and the assumptions given on the page.

Source keys N01, N02, and so on refer to the original PDF files listed near the end of this chapter. Source page numbers count PDF pages. CED means the supplied 2026 AP Physics 2 Course and Exam Description.

The battery as a person lifting water

A person lifts water from a low pool to a high pool. Each kilogram raised through the same height gains the same gravitational potential energy per unit mass. With the valve open, water flows down the rough vertical pipe and returns to the low pool. Viscous dissipation transfers the gravitational energy to thermal energy.



The person supplies energy, like a battery. The valve is an on/off switch. The rough vertical pipe dissipates gravitational potential energy as thermal energy, like a resistor.

RELATIONSHIP AND CONDITIONS

Water: $\Delta U_g/m = g\Delta h$, measured in J/kg

Electric source: $\varepsilon = W_{\text{source}}/q$, measured in J/C = V

A 1.5 V ideal battery supplies 1.5 J of energy per coulomb transported through its internal source process. Two coulombs receive 3.0 J. The voltage is the energy supplied per charge, not the total energy and not the amount of charge flowing each second.

EMF IS AN ENERGY-PER-CHARGE QUANTITY

The historical name electromotive force does not make ε a mechanical force. Like the person maintaining a water-height difference, the battery uses chemical energy to maintain charge separation and a potential difference. The source does not create charge.

What the water analogy predicts

Water circuit	Electrical circuit	Meaning
Mass transported each second	Charge transported each second	Current $I = \Delta Q / \Delta t$
Energy gained per kilogram	Energy supplied per coulomb	Height energy $g\Delta h$ corresponds to emf ϵ
Valve: open / closed	Switch: closed / open	Allows / stops steady flow; valve OPEN corresponds to switch CLOSED
Rough vertical pipe	Resistor	Dissipates energy as heat; water and charge are conserved
Water loses height energy on return	Charge transfers energy in a load	Energy moves to the environment; water/charge is conserved
Person supplies power	Battery supplies power	Power = energy per transported amount $\times$ transport rate

RELATIONSHIP AND CONDITIONS

$$P_{\text{source}} = \epsilon I = (J/C)(C/s) = J/s$$

Example: a 1.5 V ideal cell supplies 0.20 A. Each coulomb gains 1.5 J; 0.20 C passes each second. The source transfers 0.30 J each second, or 0.30 W.

Analogy limits: water moves because of gravity, while circuit charge responds to electric fields. In a metal, electrons drift opposite conventional current. A battery is not literally a bucket carrying positive charges through a metal, and water height is not electric potential in SI units.

PAUSE AND EXPLAIN

An open valve permits water flow, just as a closed electrical switch permits current. A closed valve stops steady flow, just as an open switch stops current. With the valve fully open and the height difference fixed, greater pipe resistance reduces the flow rate; it does not change $g\Delta h$ per kilogram. Likewise, at fixed ideal-source voltage, larger load resistance reduces current without changing the energy supplied per coulomb.

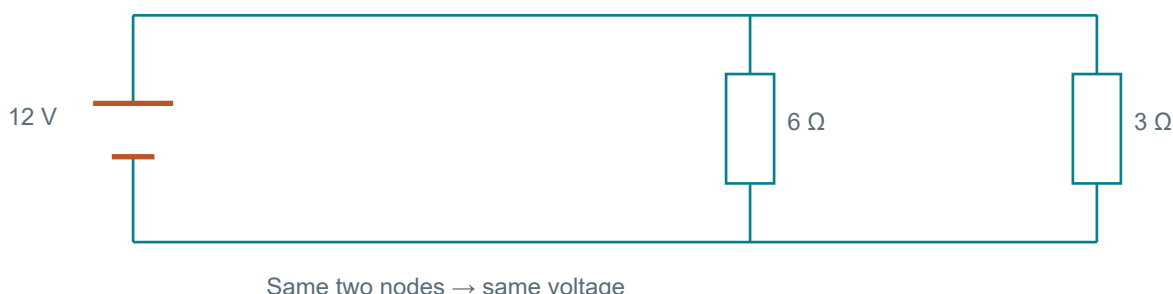
Current is charge per unit time

RELATIONSHIP AND CONDITIONS

$$I = \Delta Q / \Delta t; 1 \text{ A} = 1 \text{ C/s}$$

Conventional current points in the direction positive charge would move. Electron drift in a metal is opposite that direction.

A current requires net transport of charge. Individual electrons can have thermal motion even when there is no current; random motions average to zero net charge flow. A potential difference establishes an electric field in a resistive conductor and produces a small directed drift superimposed on that motion. Current is a signed rate associated with a chosen branch direction, not a spatial vector added by the parallelogram rule.



The two resistors connect across the same pair of nodes. Both have the battery's voltage, while the branch currents need not be equal.

A node is a set of points joined by ideal wire, all at the same potential. A branch is a path between nodes containing one or more components. A loop is a closed route. Redrawing a wire without changing its connections does not change the circuit. Label nodes before deciding whether components are in series or parallel.

Worked example

A steady current of 0.25 A flows for 40 s. The transferred charge is $Q = It = \mathbf{10 \text{ C}}$, corresponding to $\mathbf{6.25 \times 10^{19} \text{ electrons}}$ passing a cross-section. These electrons are not used up by the resistor; energy is transferred from the electrical system to the resistor and surroundings.

Read and draw a circuit schematic

A schematic preserves electrical connections rather than the physical appearance of apparatus. Trace uninterrupted conducting paths first, then mark branches and nodes. A component shared by two loops is still one physical component.



Battery



Resistor



Capacitor



Open switch



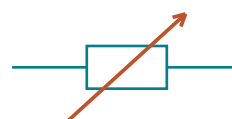
Lamp / bulb



Ammeter



Voltmeter



Variable resistor

Common circuit symbols. A rectangular resistor is used here; a zigzag resistor is also conventional. The longer battery line marks the positive terminal. A diagonal arrow indicates an adjustable component.

Drawing feature	Interpretation
Filled dot at a junction	The connected wires share an electrical node.
Crossing with a bridge or no junction mark	Usually no connection; follow the diagram's stated convention.
Ideal wire bent into a new shape	Connectivity and circuit behavior are unchanged.
Two elements in a single unbranched path	Same current: a series connection.
Both ends of two elements join the same two nodes	Same potential difference: a parallel connection.

A reliable redrawing method

Give each node a letter. List the two nodes joined by each component. Draw a simpler arrangement that preserves this list. Visual closeness on the page does not make components parallel, and a shared wire alone does not make them series.

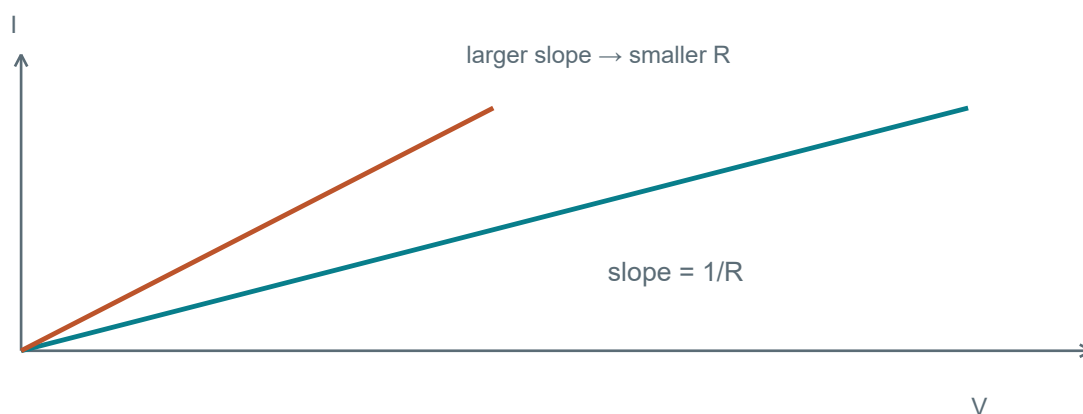
Resistance, resistivity and graphs

RELATIONSHIP AND CONDITIONS

$R = \rho L/A$; $\Delta V = IR$ for an ohmic element

R : Ω ; resistivity ρ : $\Omega \cdot \text{m}$. In a cylindrical wire, $A = \pi r^2$.

Resistance describes a particular object. Resistivity describes its material in a specified state, including temperature. A longer wire creates more opposition; a larger cross-section provides more parallel paths for charge. Do not confuse radius with area: doubling radius multiplies area by four and divides resistance by four if all other properties stay fixed.



For I plotted vertically versus V horizontally, the slope is $1/R$. For V versus I , the slope is R . Always read the axes before using a slope.

An ideal ohmic resistor has a linear I - V relation through the origin with constant resistance in the model. Real metallic resistivity usually rises as temperature rises. A filament bulb heats as current rises and can therefore have a curved I - V relation. “Ohmic” should not be used to claim that every real metal’s resistance is independent of temperature.

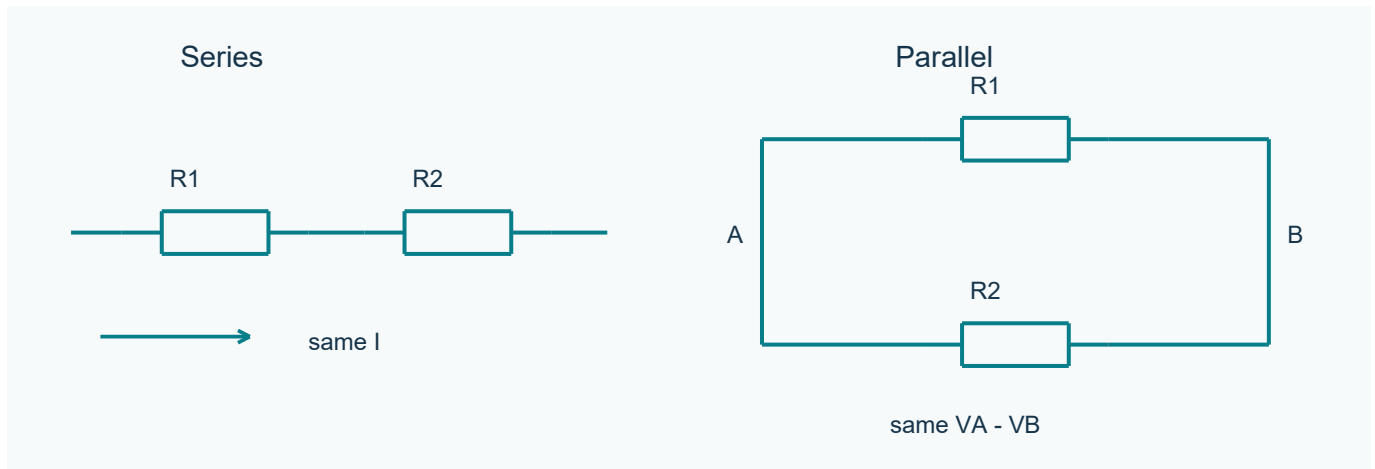
Worked example

A uniform wire has $L = 2.0 \text{ m}$, $A = 0.50 \text{ mm}^2 = 5.0 \times 10^{-7} \text{ m}^2$ and $\rho = 1.7 \times 10^{-8} \Omega \cdot \text{m}$. $R = \mathbf{0.068 \Omega}$.

Stretch it to twice its length while keeping volume and resistivity fixed: area halves, so R becomes $\mathbf{4R = 0.272 \Omega}$.

Series: the original voltage-sharing examples

Every charge that passes one series resistor must pass the other. Both resistors therefore carry the same current. The source energy per coulomb is shared between the resistor drops.



Left: series resistors share current. Right: parallel branches share the potential difference between nodes A and B.

RELATIONSHIP AND CONDITIONS

Series: $I_1 = I_2 = I$

$V = IR_1 + IR_2$; $R_{eq} = R_1 + R_2$

$V_1/V_2 = R_1/R_2$

Original numerical pair: put $3\ \Omega$ and $7\ \Omega$ in series across 100 V . $I = 100/10 = 10\text{ A}$; the drops are 30 V and 70 V . Replace $7\ \Omega$ with $2\ \Omega$: I becomes $100/5 = 20\text{ A}$, and the drops become 60 V and 40 V .

THE COMPARISON

The $3\ \Omega$ resistor did not change, but its voltage did because changing the other resistor changed the loop current. “The voltage divides in the resistance ratio” is useful only after identifying the common current.

Parallel: voltage is shared and current adds

RELATIONSHIP AND CONDITIONS

$$V_1 = V_2 = V$$

$$I = I_1 + I_2 = V/R_1 + V/R_2$$

$$1/R_{eq} = 1/R_1 + 1/R_2$$

Original configuration	Equivalent resistance	Reason
Two 10 Ω resistors in parallel	5 Ω	$1/R_{eq} = 1/10 + 1/10$
Three 6 Ω resistors in parallel	2 Ω	$1/R_{eq} = 3/6$
N identical resistors R in parallel	R/N	N identical current paths share V
Two unequal parallel resistors	$1/I_2 = R_2/R_1$	Current is inversely proportional to R at fixed V

Adding a parallel branch to an ideal battery increases total current. It does not force an unchanged total current to divide between more branches. The existing branch currents stay unchanged if their resistances and the battery voltage stay fixed.

PAUSE AND EXPLAIN

Two identical bulbs are lit in parallel on an ideal battery. A third identical branch is added. What changes? Total source current and power increase. Each original bulb has the same voltage and therefore the same power in the constant-resistance model.

A real battery can lose terminal voltage as total current rises. That later refinement changes the prediction for the original branches.

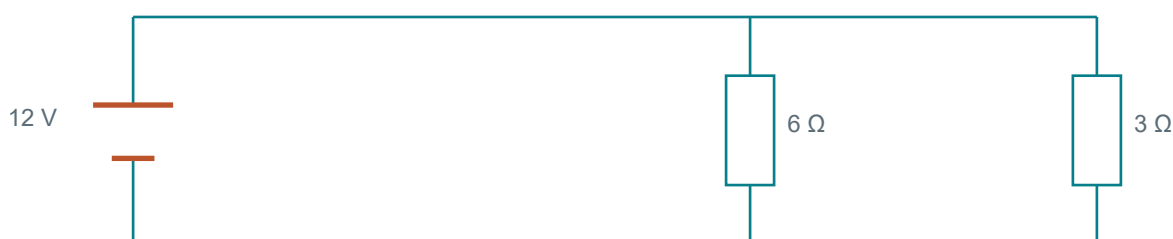
Series and parallel resistor networks

RELATIONSHIP AND CONDITIONS

Series: $R_{\text{eq}} = R_1 + R_2 + \dots$ (same current)

Parallel: $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \dots$ (same voltage)

Two resistors are in series only if the junction between them has no alternative current path. They are in parallel only if both endpoints share the same two nodes. A visually vertical or horizontal arrangement proves neither condition. A bridge circuit may not reduce to a simple series-parallel combination; use node/loop equations instead.



Same two nodes $\rightarrow$ same voltage

A $6\ \Omega$ and $3\ \Omega$ parallel pair has $R = 2\ \Omega$. Adding branches lowers equivalent resistance because the same applied voltage produces a greater total current.

Worked example: reduce, then expand

A $4.0\ \Omega$ resistor is in series with a parallel combination of $6.0\ \Omega$ and $3.0\ \Omega$ on $12\ \text{V}$. First $R_{\text{parallel}} = 2.0\ \Omega$, so total $R = 6.0\ \Omega$ and $I = 2.0\ \text{A}$. The $4\ \Omega$ drop is **8.0 V**, leaving **4.0 V** across each branch. Branch currents are **$2/3\ \text{A}$** and **$4/3\ \text{A}$** . They sum to $2.0\ \text{A}$.

For two parallel resistors, $I_1/I_2 = R_2/R_1$. For two series resistors, $V_1/V_2 = R_1/R_2$. Derive these ratios from the shared quantity rather than memorizing an unlabeled current-divider rule.

Power: choose the quantity that is fixed

RELATIONSHIP AND CONDITIONS

$$P = I\Delta V = I^2R = (\Delta V)^2/R \text{ for a resistor}$$

Energy transferred in a constant-power interval: $E = P\Delta t$. $1 \text{ W} = 1 \text{ J/s}$.

The energy transferred per unit charge is the voltage drop; the charge transferred per unit time is current. Their product is power. Resistive elements transfer electrical energy into thermal energy and sometimes light. Bulb brightness is usually compared through electrical power for identical bulbs, while remembering that real filament resistance can change with temperature.

Constraint	Useful equation	Effect of increasing R
Fixed current	$P = I^2R$	Power increases.
Fixed voltage	$P = V^2/R$	Power decreases.
Two identical bulbs in series	Same current	Same power; dimmer than one alone on the same ideal battery.
Two identical bulbs in parallel	Same voltage	Each matches one alone on the same ideal battery.

Worked example: compare connections

Two $6.0 \, \Omega$ resistors use a 12 V ideal battery. **Series:** $R_{\text{eq}} = 12 \, \Omega$, $I = 1.0 \text{ A}$, each $P = 6.0 \text{ W}$; total 12 W . **Parallel:** each has 12 V , $I = 2.0 \text{ A}$, $P = 24 \text{ W}$; total current 4.0 A and total power 48 W .

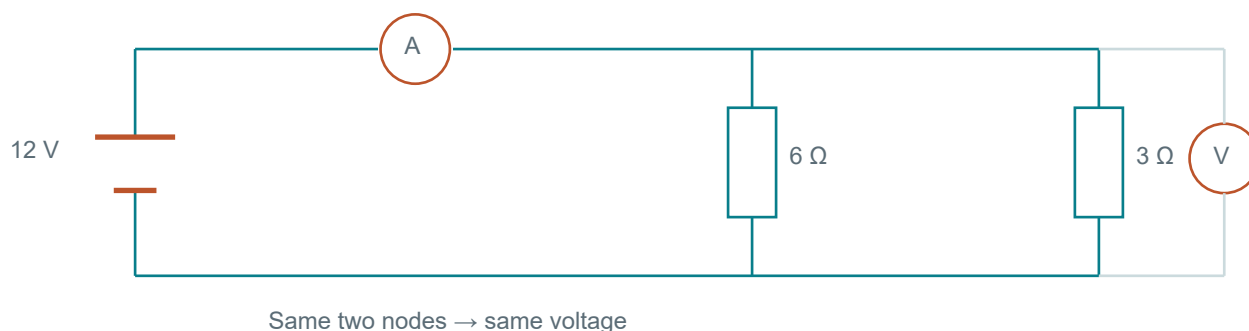
Conservation of energy provides a numerical check: power supplied by the source equals total power dissipated plus the rate at which energy is stored in components. In a purely resistive steady circuit, $\Sigma P_{\text{resistors}} = \mathcal{E}I$ for an ideal battery. In an RC transient, some of the source power increases stored capacitor energy.

Common trap

A larger resistance does not by itself determine brightness. State what stays fixed. Adding a parallel branch increases total current from an ideal battery; it does not divide a fixed total current unless a current source is specified.

Open circuits, short circuits and meters

Ideal element or condition	Model	Reasoning consequence
Wire / closed switch	Negligible resistance	Same potential along connected wire.
Open switch	No conducting path	No steady current across the opening; voltage may be nonzero.
Short across a component	Zero-resistance bypass	The bypass imposes zero voltage across that component.
Ammeter	Zero resistance, in series	Measures branch current without an ideal voltage drop.
Voltmeter	Infinite resistance, in parallel	Measures a potential difference without ideal current draw.



The ammeter lies in the main current path; the voltmeter bridges two nodes. In the ideal model neither changes the circuit it is intended to measure.

A real ammeter adds series resistance and can reduce the current. A real voltmeter draws some parallel current and can change the voltage being measured. The size and direction of the resulting change depend on the rest of the circuit. Connecting an ideal ammeter directly across a battery creates a short, so the ideal model would predict an unbounded current; real internal and wire resistance limit it.

Checkpoint

An open switch has $I = 0$. Does it necessarily have $\Delta V = 0$? **No**. It can support the full battery voltage. Conversely, an ideal wire can carry current while its potential drop is zero.

A real battery has internal resistance

RELATIONSHIP AND CONDITIONS

Discharging battery: $V_{\text{terminal}} = \mathcal{E} - Ir$

With external resistance R : $I = \mathcal{E}/(R + r)$

Power balance: $\mathcal{E}I = I^2R + I^2r$.

The emf $\mathcal{E}$ is the energy supplied by the battery's internal processes per unit charge. It is a voltage, despite the name "electromotive force." Model the battery as an ideal source in series with an internal resistor r . Its terminal voltage equals $\mathcal{E}$ at zero current and falls below $\mathcal{E}$ while delivering current. A voltmeter across the disconnected battery approximates this open-circuit emf.

Worked example

A 12.0 V battery with $r = 1.0 \, \Omega$ drives $R = 5.0 \, \Omega$. $I = \mathbf{2.0 \, A}$; terminal voltage = **10.0 V**. The load receives **20 W**, while **4.0 W** is dissipated internally. The chemical source supplies **24 W**.

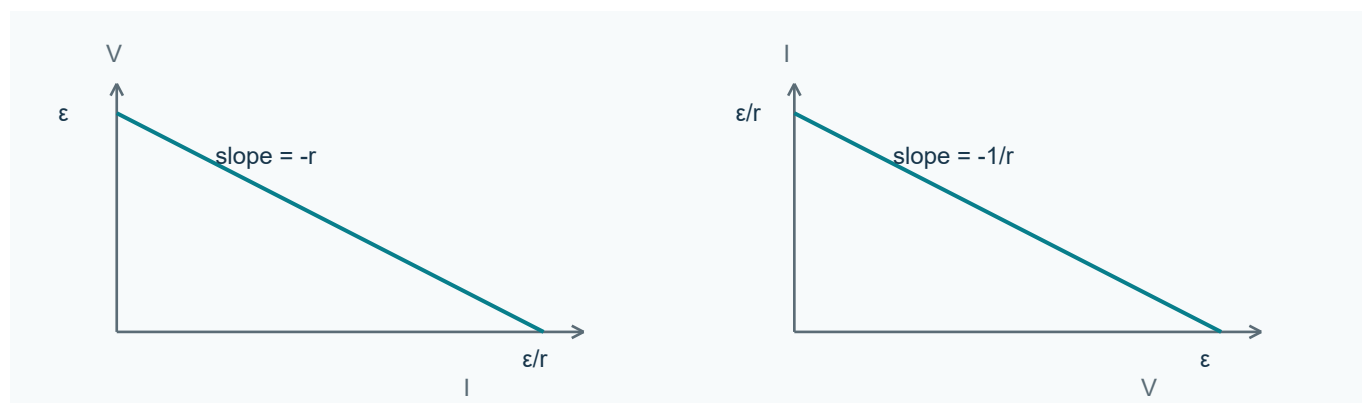
Plot terminal voltage V vertically against discharge current I horizontally: the intercept is $\mathcal{E}$ and the slope is $-r$. Do not call a reduced terminal voltage evidence that the battery's emf itself changed during one idealized measurement. Conversely, real batteries can change emf with state of charge, so the model assumes stable conditions over the experiment.

Extension: a charging battery

When current is forced into the positive terminal, the external terminal voltage must exceed the emf in the simple model: $V_{\text{terminal}} = \mathcal{E} + Ir$, with I now the positive charging-current magnitude. The minus-sign discharge formula cannot be reused without defining current direction.

Do not connect ideal batteries of unequal emf directly in parallel. The ideal constraints conflict, and the supplied CED excludes this assessment case. With real sources, internal resistance and additional physics determine the circulating currents.

Battery graphs: read the axes first



The same battery model on two sets of axes. Swapping the axes changes the slope from $-r$ to $-1/r$.

Graph axes	Equation	Slope	Intercept
V vertical, I horizontal	$V = \epsilon - rI$	$-r$	V-intercept ϵ
I vertical, V horizontal	$I = \epsilon/r - V/r$	$-1/r$	I-intercept ϵ/r

Worked comparison: a line $V = 1.50 - 0.80I$ in SI units means $\epsilon = 1.50 \text{ V}$ and $r = 0.80 \Omega$. If plotted as I versus V instead, the slope is -1.25 A/V and the vertical intercept is 1.875 A .

THE SOURCE QUESTION ABOUT A SMALLER r

For an I -versus- V graph with the same ϵ and smaller r , both the magnitude of the negative slope and the vertical intercept increase. The horizontal intercept stays ϵ . For V versus I , the vertical intercept stays ϵ and the slope becomes less steep.

The large-current intercept is an extrapolation of a simple model. It is not a recommendation to short-circuit a real battery.

Kirchhoff: charge conservation before loop signs

Rule	Physical conservation	Equation
Junction	Charge does not accumulate indefinitely at a node	$\Sigma I_{in} = \Sigma I_{out}$
Loop	Energy per unit charge returns to its starting value	$\Sigma \Delta V = 0$

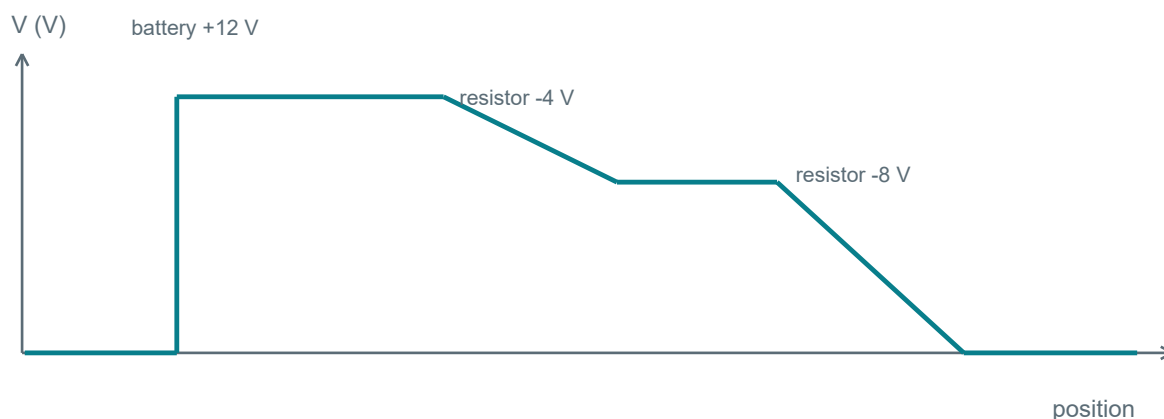
Follow the original order: first use the junction rule to relate branch currents, then apply loop equations. Current arrows are initial choices. A negative answer later means that the true current runs opposite the chosen arrow.

RELATIONSHIP AND CONDITIONS

Across a resistor with the assigned current: $\Delta V = -IR$

Across a resistor against the assigned current: $\Delta V = +IR$

Across a source from - to +: $\Delta V = +\epsilon$

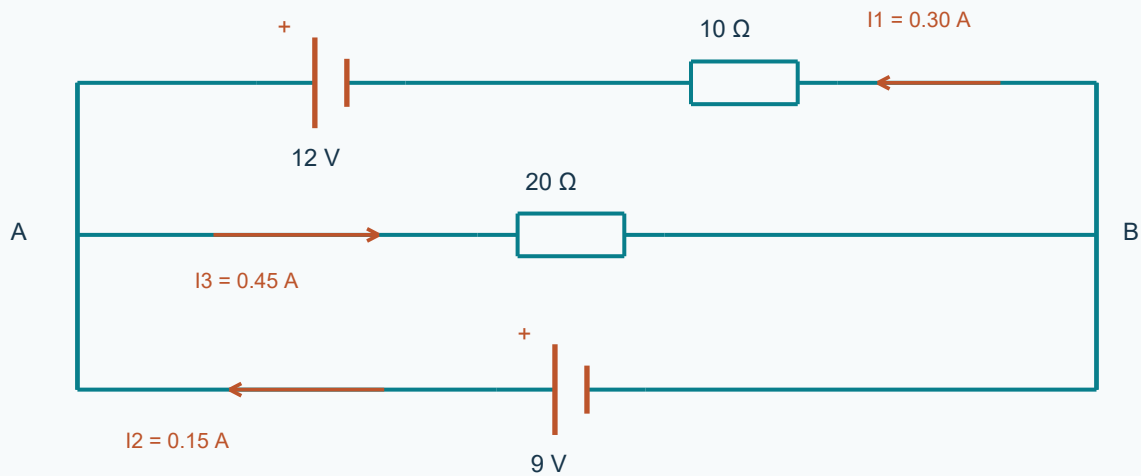


A potential walk rises in the source and falls in the resistors, returning to its starting potential.

PAUSE AND EXPLAIN

Why is current not used up by a resistor? The same charge enters and leaves in steady state. Each coulomb transfers energy, so the potential changes even though the current does not.

The original two-loop example: negative current is useful



The lower 9 V source fixes $V_A - V_B = 9$ V. The upper branch contains the 12 V source and 10 Ω resistor. The middle resistor is 20 Ω .

RELATIONSHIP AND CONDITIONS

Choose I_1 into A from the upper branch, I_2 into A from the lower branch, and I_3 from A to B through 20 Ω .

$$I_1 + I_2 = I_3$$

Lower loop: $9 - 20I_3 = 0$, so $I_3 = 0.45$ A.

Upper loop: $12 - 20I_3 - 10I_1 = 0$, so $I_1 = 0.30$ A.

Then $I_2 = 0.45 - 0.30 = 0.15$ A.

Choose a second set of arrows: J_3 points from B to A through the middle branch, and J_2 points from A to B through the lower branch. Keep I_1 entering A along the upper branch. The junction rule becomes $I_1 + J_3 = J_2$. The same physical currents are $J_3 = -0.45$ A and $J_2 = -0.15$ A; both negative signs mean that the chosen arrows oppose the actual currents.

POWER CHECK

Sources supply $12(0.30) + 9(0.15) = 4.95$ W. Resistors dissipate $(0.30)^2(10) + (0.45)^2(20) = 0.90 + 4.05 = 4.95$ W. Both current descriptions represent the same energy balance.

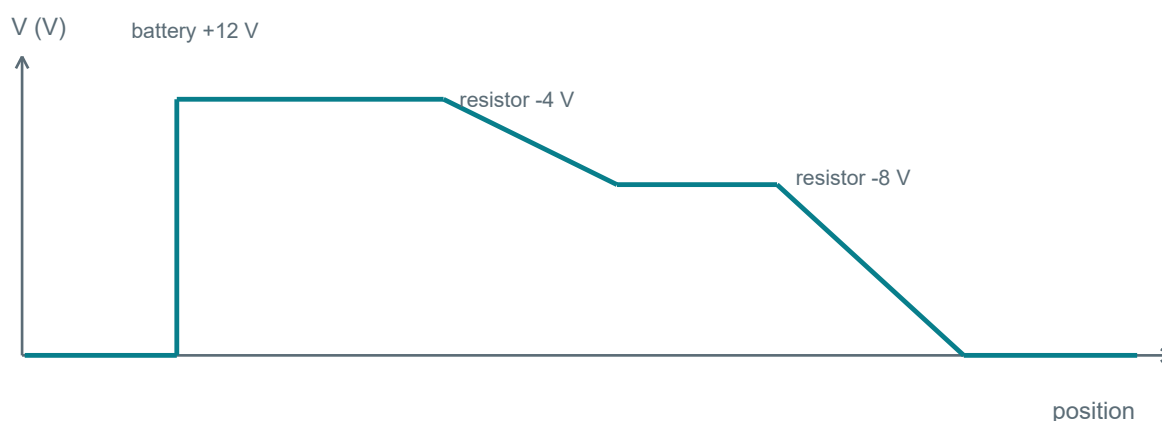
Kirchhoff's loop rule

RELATIONSHIP AND CONDITIONS

$\Sigma \Delta V$ around a closed loop = 0

Battery: - terminal $\rightarrow$ + terminal gives $+\epsilon$.

Resistor: traverse with assigned current $\rightarrow -IR$; against current $\rightarrow +IR$.



This potential walk rises by 12 V across the battery and drops by 4 V and 8 V across two resistors. It returns to its starting potential.

Choose a loop direction and record each signed potential change in order. Reversing the loop direction reverses every sign and produces the same physical solution. A resistor is always dissipative even when its voltage term is positive along your chosen traversal: that sign reflects your traversal direction, not energy creation.

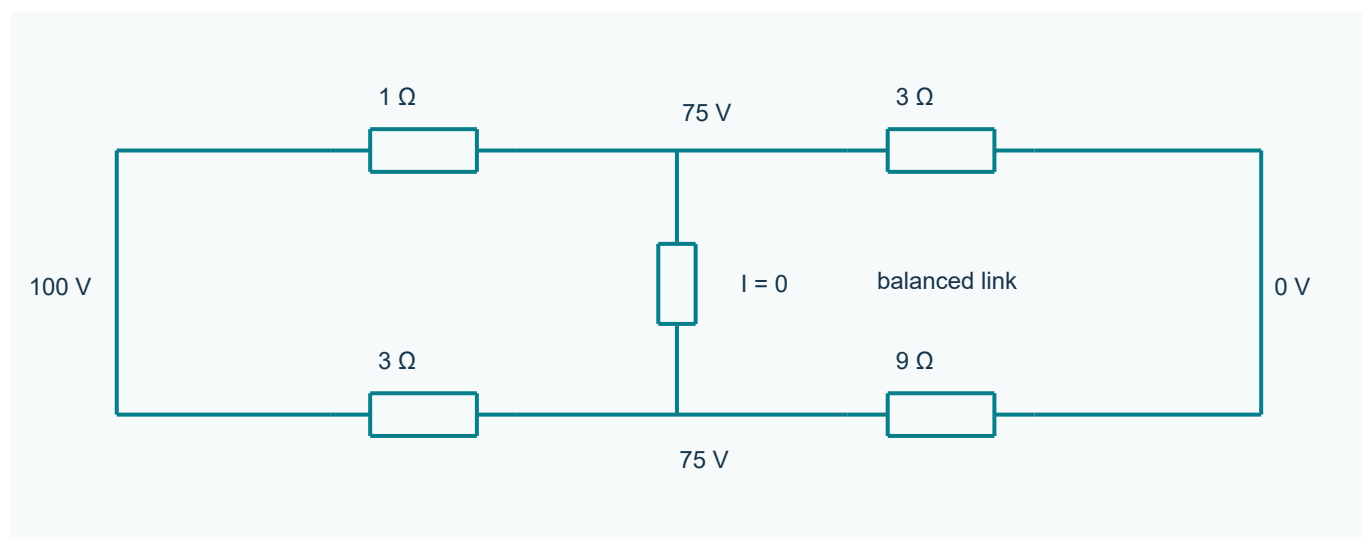
Worked example: opposing batteries

Two ideal batteries of 12 V and 4 V oppose one another in a single loop containing $2\ \Omega$ and $6\ \Omega$ resistors. Assign current in the direction driven by 12 V. The equation is $12 - 2I - 6I - 4 = 0$, so **$I = 1.0\ \text{A}$** . If the second battery aided the first, its term would be $+4\ \text{V}$ and I would be $2.0\ \text{A}$.

Scope of the model

For the ordinary lumped DC circuits here, loop voltage accounting expresses conservation of energy. If a changing external magnetic flux induces an emf, include that emf consistently; do not apply an electrostatic-only loop statement while ignoring induction.

The balanced bridge: equal potentials stop branch current



Original values: upper branch 1 Ω then 3 Ω , lower branch 3 Ω then 9 Ω , with a 100 V source across both branches.

Choose the right-hand node as 0 V and the left-hand node as 100 V. The upper current is $100/(1 + 3) = 25$ A, so the upper midpoint is $100 - 25(1) = 75$ V. The lower current is $100/(3 + 9) = 25/3$ A, so its midpoint is also $100 - (25/3)(3) = 75$ V.

RELATIONSHIP AND CONDITIONS

$$\Delta V_{\text{midpoints}} = 75 - 75 = 0$$

Any finite-resistance connecting branch carries $I = 0$.

The middle bulb is dark even though both midpoints are at a high potential relative to the chosen zero. Current through the bulb depends on the difference between its terminals. Adding the balanced link does not change the branch currents in the ideal network.

GENERAL CONDITION

A bridge balances when the resistance ratio on the upper branch equals that on the lower branch:

$R_{\text{left,top}}/R_{\text{right,top}} = R_{\text{left,bottom}}/R_{\text{right,bottom}}$. Here $1/3 = 3/9$. Compare ratios or midpoint potentials, not visual symmetry.

Capacitors in series and parallel

RELATIONSHIP AND CONDITIONS

Parallel: $C_{eq} = \Sigma C_i$ (same voltage)

Series: $1/C_{eq} = \Sigma 1/C_i$ (equal charge magnitude for initially neutral intermediate conductors)

For parallel capacitors, total delivered charge is the sum of individual plate charges. At a common V , $Q_{total} = \Sigma C_i V$, giving the sum rule. For capacitors in series, a neutral isolated intermediate conductor cannot acquire net charge; its opposite faces acquire equal and opposite charges, giving the same charge magnitude on the adjacent capacitors.

Connection	Shared quantity	Equivalent capacitance	Useful check
Parallel	Voltage	Sum	Larger than any component.
Series	Charge magnitude*	Reciprocal sum	Smaller than any component.

*If intermediate conductors initially carry nonzero net charge, a more general charge-conservation analysis is needed. Equal charge is the standard initially uncharged-series model, not an unconditional statement about any precharged network.

Worked example

Connect $3.0 \mu\text{F}$ and $6.0 \mu\text{F}$ in series across 12 V . $C_{eq} = 2.0 \mu\text{F}$, so each capacitor carries **$24 \mu\text{C}$** . Voltages are **8.0 V** and **4.0 V** . Energies are **$96 \mu\text{J}$** and **$48 \mu\text{J}$** ; total $144 \mu\text{J} = \frac{1}{2} C_{eq} V^2$. In parallel, $C_{eq} = 9.0 \mu\text{F}$ and both capacitors have 12 V .

Checkpoint

In a series pair, the smaller capacitor has the larger voltage because $V = Q/C$ and the charge magnitudes match. Equal charge does not mean equal voltage.

Resistors and capacitors: paired connection rules

Connection	Resistors	Capacitors
Series constraint	Same current	Equal plate-charge magnitudes for neutral intermediate nodes
Series combination	$R_{eq} = R_1 + R_2 + \dots$	$1/C_{eq} = 1/C_1 + 1/C_2 + \dots$
Parallel constraint	Same voltage	Same voltage
Parallel combination	$1/R_{eq} = 1/R_1 + 1/R_2 + \dots$	$C_{eq} = C_1 + C_2 + \dots$
N identical units in series	$R_{eq} = NR$	$C_{eq} = C/N$
N identical units in parallel	$R_{eq} = R/N$	$C_{eq} = NC$

The original notes place these rules side by side. Derive each from its shared quantity: series resistors add voltage drops at the same current, while parallel capacitors add plate charge at the same voltage. The formulas reverse because the defining relationships differ.

PAUSE AND EXPLAIN

Why does the smaller capacitor in a series pair have the larger voltage? Equal Q and $V = Q/C$ give the result. Why does the larger resistor in a series pair have the larger voltage? Equal I and $V = IR$ give that result.

Charging and discharging an RC circuit

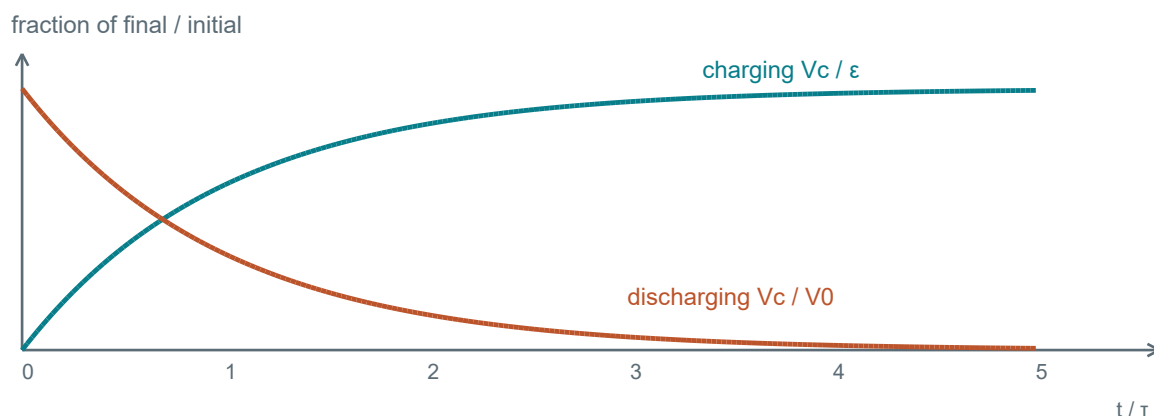
CED boundary: initial/final states, qualitative curves and time-constant meaning are core. The exponential formulas below are a mathematical extension retained to explain the curves; time-dependent algebraic modeling is not required by the supplied CED.

RELATIONSHIP AND CONDITIONS

Simple series RC circuit: $\tau = RC$

Charging from $Q = 0$: $Q = C\epsilon(1 - e^{-t/\tau})$, $I = (\epsilon/R)e^{-t/\tau}$

Discharging: $Q = Q_0 e^{-t/\tau}$, $|I| = (V_0/R)e^{-t/\tau}$



At $t = \tau$, a charging capacitor reaches 63.2% of its final voltage; a discharging capacitor retains 36.8% of its initial voltage. Exact completion is asymptotic.

Immediately after connection, an initially uncharged capacitor has zero voltage and can be modeled as a wire for finding the initial current. After a long time on a DC source, its current tends to zero and that branch acts as an open circuit. A precharged capacitor does not start as a zero-voltage wire: its initial voltage must be retained. The capacitor's voltage is continuous through switching in a finite-resistance circuit.

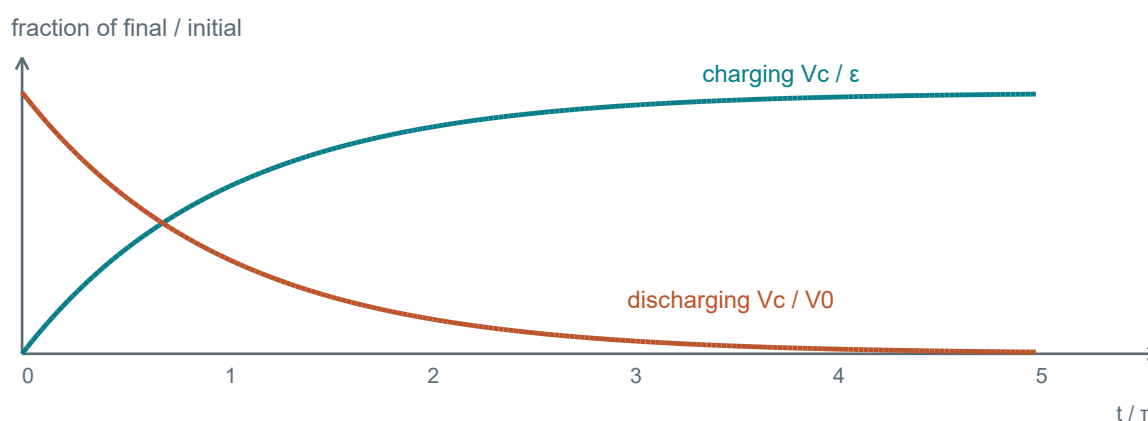
Worked example

For $R = 100 \text{ k}\Omega$, $C = 20 \text{ }\mu\text{F}$ and $\epsilon = 12 \text{ V}$, $\tau = 2.0 \text{ s}$. Initial current = **120 μA** . At $t = 2.0 \text{ s}$, $V_C = 7.59 \text{ V}$, $Q = 152 \text{ }\mu\text{C}$, and $I = 44.1 \text{ }\mu\text{A}$. At long times, $Q \rightarrow 240 \text{ }\mu\text{C}$ and $I \rightarrow 0$.

RC curves: one time constant and two limiting circuits

Quantity	Just after closing, uncharged C	Long after charging	At one τ
Capacitor voltage V_C	0	ϵ in simple series RC	0.632ϵ
Resistor voltage V_R	ϵ	0	0.368ϵ
Current I	ϵ/R	0	$0.368 I_0$
Capacitor charge Q	0	$C\epsilon$	$0.632 Q_{\text{final}}$

For discharge through a resistor, Q , V_C and current magnitude all retain about 37% of their initial values after one $\tau = RC$. A larger R or C gives a longer time scale. Compare initial and final values separately from how quickly the curve moves between them.

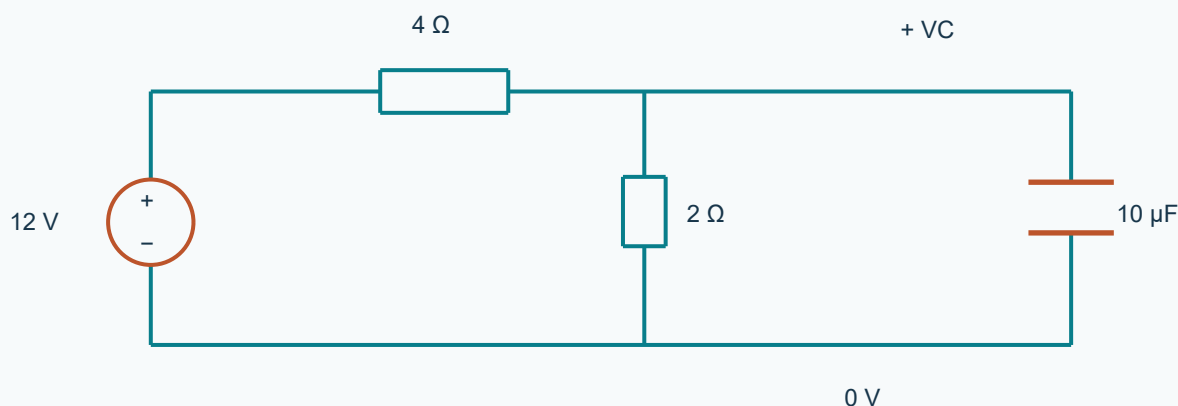


Charging rises toward its final value; discharging falls toward zero. The endpoints are approached asymptotically.

INITIAL VOLTAGE MATTERS

“A capacitor initially acts like a wire” applies to an initially uncharged capacitor in this model. A precharged capacitor preserves its nonzero voltage at the switching instant. After a long time, its branch carries no DC current, but its terminal voltage need not equal the full battery emf.

RC reasoning beyond one memorized formula



An initially uncharged capacitor is connected to the 12 V source at $t = 0$. The $2\ \Omega$ resistor remains in parallel with the capacitor.

For a complex switched network, redraw it twice: once at the instant after switching with each capacitor's initial voltage, and once at long times with each capacitor branch open. Calculate node potentials in each redraw. The voltage of a fully charged capacitor equals the voltage between its own two terminals, which need not equal the battery emf.

Worked steady-state example

A 12 V battery feeds a $4\ \Omega$ resistor followed by a node connected to ground through a $2\ \Omega$ resistor. A capacitor is connected from that node to ground. Long after charging, the capacitor draws no current. The resistor current is $12/(4 + 2) = 2\ \text{A}$, so $V_{C,\text{final}} = 4\ \text{V}$. If $C = 10\ \mu\text{F}$, $Q_{\text{final}} = 40\ \mu\text{C}$.

The time constant uses the effective resistance seen by the capacitor. In the example, set the ideal voltage source to zero (replace it with a wire) and look between the capacitor terminals. The $4\ \Omega$ and $2\ \Omega$ resistors are then parallel, giving $R_{\text{effective}} = 4/3\ \Omega$. Thus $\tau = (4/3\ \Omega)(10\ \mu\text{F}) = 13.3\ \mu\text{s}$. This reduction is a useful extension for more complicated single-capacitor circuits.

RELATIONSHIP AND CONDITIONS

Stored energy: $U_C(t) = \frac{1}{2}CV_C(t)^2$

For discharge, $U_C/U_0 = e^{-2t/\tau}$. Energy falls faster than charge.

Charging an initially uncharged capacitor from a fixed ideal source through resistance draws total source energy $C\varepsilon^2$; the final stored energy is $\frac{1}{2}C\varepsilon^2$. The other half becomes thermal energy in the resistance. Making the resistance smaller shortens the process but does not eliminate this total dissipation in that idealized charging model.

Two circuit experiments

A | Measure battery emf and internal resistance

Connect a variable load and an ammeter in series with the battery; connect a voltmeter across the battery terminals. Collect at least five stable (I, V) pairs over a modest current range, repeating readings. Plot V against I . Fit a straight line: intercept = $\mathcal{E}$ and slope = $-r$. Keep the battery's state and temperature approximately constant; avoid prolonged large currents that change the source during the test.

Graph interpretation

Suppose $V = 1.50 \text{ V} - (0.80 \, \Omega)I$. The emf is **1.50 V** and internal resistance is **0.80 Ω** . The slope is negative because greater current creates greater internal voltage drop. An ammeter's resistance changes the operating current; using the measured current still permits this terminal-voltage analysis if meter loading is otherwise negligible.

B | Determine an RC time constant

Discharge the capacitor through a known resistor while a voltage sensor records V_C . Measure the time to $0.368V_0$, or plot $\ln(V_C/V_0)$ against time: slope = $-1/\tau$. The logarithmic method is an analysis extension; the 37% method directly expresses the time-constant definition. Repeat runs and check that the initial voltage is reproducible.

A voltmeter of finite input resistance adds a parallel discharge path and makes the observed decay faster than predicted using the intended resistor alone. Leakage can have a similar effect. A sampling interval comparable to τ misses the curve; choose a much shorter interval. Unknown switch timing shifts the apparent origin. State which effect your improvement addresses.

Student practice

1. A 0.40 A current flows for 25 s. How much charge passes? How many electrons is that?
2. A wire's length and radius both double while its material and temperature stay fixed. What happens to resistance?
3. A 12 V battery drives 2 Ω in series with a parallel pair 3 Ω and 6 Ω . Find total current, each branch current and total power.
4. A battery has $\epsilon = 9.0$ V and $r = 0.50$ Ω . With a 4.0 Ω load, find current, terminal voltage and internal heating power.
5. Two equal resistors R are connected first in series, then in parallel across the same ideal voltage V. Find the ratio $P_{\text{parallel}}/P_{\text{series}}$.

Chapter review: capacitors and graphs

6. A junction has 2.0 A and 1.5 A entering; 0.80 A and an unknown current leave. Find the unknown current.
7. Capacitors 2.0 μF and 3.0 μF are in series across 10 V. Find equivalent capacitance, charge and individual voltages.
8. Extension: A 50 k Ω resistor charges a 40 μF capacitor from a 6.0 V source. Find τ , initial current and voltage at $t = \tau$.
9. Extension: A capacitor discharges for two time constants. What fractions of its initial charge and energy remain?
10. A V-versus-I graph for a battery has intercept 10.0 V and slope -2.0 V/A. Determine ϵ and r . What current flows with an 8.0 Ω external load?

Practice solutions

1. $Q = It = 10 \text{ C}$; $N = Q/e = 6.25 \times 10^{19}$.

2. L doubles, A quadruples; R scales by $2/4 = 1/2$.

3. Parallel $R = 2 \Omega$; total $R = 4 \Omega$; $I = 3 \text{ A}$. The parallel section has 6 V , so its currents are 2 A and 1 A . Total power = $VI = 36 \text{ W}$.

4. $I = 9/4.5 = 2.0 \text{ A}$; $V_{\text{terminal}} = 8.0 \text{ V}$; $I^2r = 2.0 \text{ W}$.

5. $P_{\text{series}} = V^2/(2R)$; $P_{\text{parallel}} = V^2/(R/2)$. Ratio = 4.

Worked answers: capacitors and graphs

6. $2.0 + 1.5 = 0.80 + I$, so $I = 2.70 \text{ A}$ leaving.

7. $C_{\text{eq}} = 6/5 = 1.2 \text{ }\mu\text{F}$. $Q = 12 \text{ }\mu\text{C}$. V values = 6 V and 4 V ; their sum is 10 V .

8. $\tau = (50 \times 10^3)(40 \times 10^{-6}) = 2.0 \text{ s}$. $I_0 = 120 \text{ }\mu\text{A}$. $V_C(\tau) = 6(1 - e^{-1}) = 3.79 \text{ V}$.

9. Charge fraction $e^{-2} = 0.135$. Energy fraction $e^{-4} = 0.0183$, because energy is proportional to Q^2 .

10. $\mathcal{E} = 10.0 \text{ V}$, $r = 2.0 \text{ }\Omega$. $I = 10/(8 + 2) = 1.0 \text{ A}$.

Original notes and syllabus map

The source sequence determines the teaching order. Repeated examples are consolidated while distinct reasoning, diagrams and comparisons are retained.

Original source	Content retained
N02, pp.4-6 Electrostatics 10.3-10.5, 10.7 + Circuits 11.1-11.3, 11.5.pdf	Water-lifting analogy; I, V, R; original series/parallel examples
N03, pp.1-6 Circuits 11.2-11.8 + Electrostatics 10.6.pdf	Power, meters, resistivity, real batteries and source combinations
N04, pp.1-9 Circuits 11.2-11.7 + Electrostatics 10.6.pdf	Junction-first Kirchhoff sequence, two-loop sign comparison and capacitors
N05, pp.1-6 Electrostatics 10.6 + Circuits 11.2-11.8.pdf	Balanced bridge; RC curves, limiting states and time constants
N29, pp.1,7-9 Problem Solving - Mixed Review 9-15 - Circuits and Electrostatics Focus.pdf	RC review and circuit experiment
N36, pp.1-7 Problem Solving - Circuits 11.2-11.3, 11.5-11.8 + Magnetism 12.1-12.4 + Electrostatics 10.6-10.7.pdf	Capacitor networks, internal-resistance graph and geometry comparisons

Curriculum: supplied *AP Physics 2 Course and Exam Description*, 2026, Unit 11. Topic labels on each content page show where the syllabus connects to the original lesson. Extensions remain labeled.

Clarifications and independent references

Point checked	Treatment in this chapter
Battery analogy	Mass flow corresponds to charge flow; energy per mass corresponds to energy per charge.
Ohmic model	Constant R describes the specified ideal model or fixed material state. Real metals can change resistivity with temperature.
Battery graph	$V(I)$ slope is $-r$, while $I(V)$ slope is $-1/r$.
RC syllabus boundary	Qualitative time behavior, τ and initial/final states are core. Exact exponentials and effective-resistance calculations are extensions.
Capacitor voltage	The long-time voltage is the difference between its own two nodes, not automatically $\mathcal{E}$.

Independent checks

OpenStax: [electromotive force](#)

OpenStax: [Ohm's law](#)

OpenStax: [RC circuits](#)

Constants and calculations use the values stated in each example. Historical practice provides context and reasoning patterns. New worked questions and corrected explanations are distinguished from an official answer key.

Master summary: constraints before formulas

Idea	Relationship	Condition / interpretation
Current	$I = \Delta Q / \Delta t$	Charge flow rate; conventional direction uses positive charge
Voltage and emf	$\Delta V = \Delta U / q$; $\varepsilon = \text{source work}/q$	Energy per charge; $1 \text{ V} = 1 \text{ J/C}$
Resistance	$R = \rho L / A$; $V = IR$	Geometry and material state; fixed R for an ohmic model
Power	$P = IV = I^2 R = V^2 / R$	State whether I or V is held fixed
Real battery	$V_{\text{terminal}} = \varepsilon - Ir$; $I = \varepsilon / (R + r)$	Discharging-source model; open-circuit $V = \varepsilon$
Junction and loop	$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$; $\Sigma \Delta V = 0$	Conservation of charge and energy
RC time constant	$\tau = RC$	63% of charging rise or 37% of discharge remains after τ
Connection	Resistors	Capacitors
Series	Same I ; $R_{\text{eq}} = \Sigma R$	Same $ Q $ in standard neutral-node model; $1/C_{\text{eq}} = \Sigma 1/C$
Parallel	Same V ; $1/R_{\text{eq}} = \Sigma 1/R$	Same V ; $C_{\text{eq}} = \Sigma C$
Initial/final RC	Use actual resistor currents in each limiting circuit	Initially uncharged: zero voltage. Long-time DC: zero branch current

Final check: conserved charge is not used up. Energy per charge changes through circuit elements. A negative solved current reverses the assumed arrow.