

Electrostatics

Keep the original sequence: force, field, potential energy, potential and electric work. Then use the conducting sphere and parallel plates to connect the representations. Capacitors retain their original energy derivation and material comparisons.

TEACHING ROUTE

The explanations follow the original lesson sequence. Connections, analogies and worked reasoning remain together. Syllabus additions appear beside the ideas they support.

Lesson group	Focus
Force and field	Charge model, Coulomb force and vector superposition
Energy and potential	Signed pair sums, energy per charge and equipotentials
Work and conductors	System boundaries, conducting spheres and field-potential graphs
Charge and capacitance	Contact, induction, capacitor geometry and stored energy
Practice and reference	Original multi-charge reasoning, experiments and a summary table

Predict first, draw the situation, then calculate. Use the class questions for discussion and the chapter review for independent practice. Unless stated otherwise, numerical examples use idealized models and the assumptions given on the page.

Source keys N01, N02, and so on refer to the original PDF files listed near the end of this chapter. Source page numbers count PDF pages. CED means the supplied 2026 AP Physics 2 Course and Exam Description.

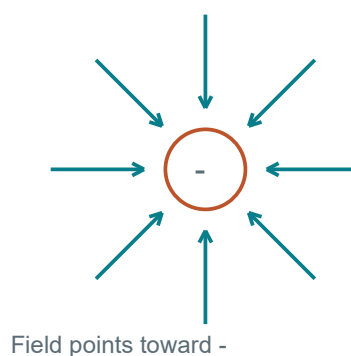
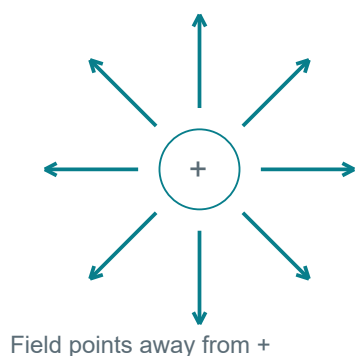
Charge and Coulomb force

Protons carry $+e$, electrons carry $-e$, and neutrons carry no net charge. Ordinary isolated objects acquire net charge in integer multiples of $e = 1.60 \times 10^{-19} \text{ C}$. Rubbing objects usually transfers electrons; it does not create charge or move protons between solid objects. A positive object has an electron deficit relative to its neutral state.

RELATIONSHIP AND CONDITIONS

$$F = k|q_1 q_2|/r^2; k = 1/(4\pi\epsilon_0) \approx 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$

Point charges in vacuum; r is their separation. Like signs repel; opposite signs attract.



The electric field is a property of the source configuration. The force on a negative test charge points opposite the drawn field.

For several charges, find each force on the selected object, resolve into components and add the components. Do not add the magnitudes unless the vectors point in the same direction. The two forces within one interacting pair have equal magnitudes, even when the objects have different masses or charge magnitudes. Different masses can still have different accelerations.

Worked example

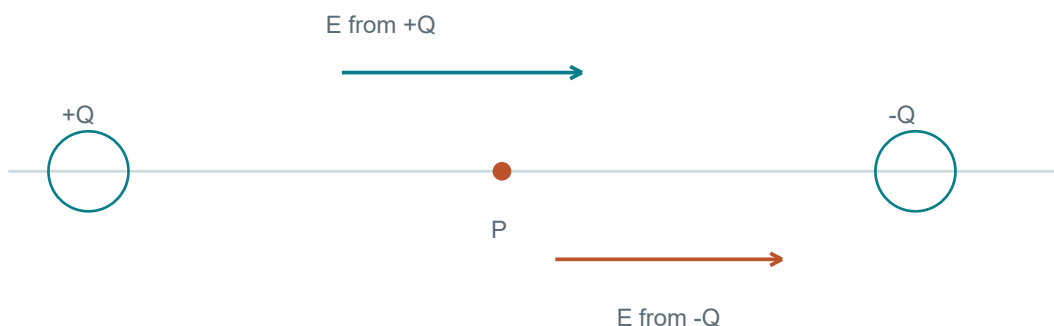
A $+2.0 \mu\text{C}$ charge and a $-3.0 \mu\text{C}$ charge are 0.30 m apart. $F = (8.99 \times 10^9)(2.0 \times 10^{-6})(3.0 \times 10^{-6})/(0.30)^2 = \mathbf{0.60 \text{ N}}$, attractive. Doubling their separation reduces the force to $\mathbf{0.15 \text{ N}}$. Each object feels the same force magnitude.

Electric field: build a vector map

RELATIONSHIP AND CONDITIONS

$$\mathbf{E} = \mathbf{F}/q_{\text{test}}; E_{\text{point}} = k|Q|/r^2; \mathbf{E}_{\text{net}} = \sum \mathbf{E}_i$$

Units: $\text{N/C} = \text{V/m}$. The test charge is small enough not to disturb the source distribution.



At the midpoint between equal opposite charges, both field contributions point from +Q toward -Q. Their potentials cancel, but their fields add.

At each location, a field arrow gives the force direction for a positive test charge. Field lines are tangent to the field, start on positive sources and end on negative sources or at infinity. Lines do not intersect because the field has a single direction at a point. Closer line spacing can represent a stronger field if the drawing uses a consistent convention. A field line is not generally the actual trajectory of a moving particle.

Worked example: two opposite sources

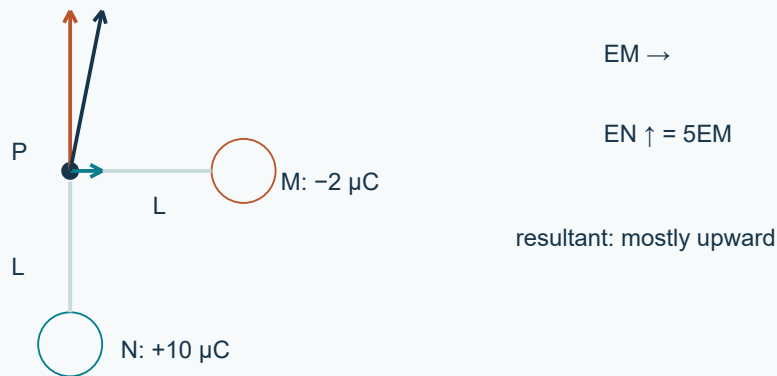
Place $+2.0 \text{ nC}$ at $x = -0.10 \text{ m}$ and -2.0 nC at $x = +0.10 \text{ m}$. At $x = 0$, each field has magnitude $(8.99 \times 10^9)(2.0 \times 10^{-9})/(0.10)^2 = 1798 \text{ N/C}$. Both point right, so **$\mathbf{E} = 3.60 \times 10^3 \text{ N/C}$ right**. An electron there feels **$5.76 \times 10^{-16} \text{ N}$ left**.

Checkpoint

At the midpoint of two equal positive charges, what cancels? **The fields cancel by symmetry; the potentials add.** Zero electric field does not imply zero electric potential.

The original vector comparison: direction matters

At point P, a negative charge M lies a distance L to the right and a positive charge N lies a distance L below. Their charges are $-2.0 \mu\text{C}$ and $+10 \mu\text{C}$. Let E_M be the magnitude of the field at P due to M.



Fields point toward the negative charge M and away from the positive charge N. This diagram uses the original perpendicular configuration.

RELATIONSHIP AND CONDITIONS

$$\begin{aligned}
 E_N &= 5E_M \\
 E_x &= E_M; E_y = 5E_M \\
 |E| &= \sqrt{26} E_M; \theta = \tan^{-1}(5) = 78.7^\circ \text{ above } +x
 \end{aligned}$$

The field is above and to the right, much nearer the vertical axis than the horizontal axis. It is not at 45° because the two component magnitudes are unequal. A negative test particle experiences a force opposite this resultant field.

PAUSE AND EXPLAIN

Why can neither $6E_M$ nor $4E_M$ be the resultant magnitude? Those arithmetic sums apply to collinear components. Here the components are perpendicular.

Potential energy belongs to a system

RELATIONSHIP AND CONDITIONS

$$U_{\text{pair}} = kq_1q_2/r; U = \sum_{\text{distinct pairs}} kq_iq_j/r_{ij}$$

Choose $U = 0$ when all particles are infinitely separated. Retain the signs of the charges.

For like charges, U is positive and decreases as they separate. For opposite charges, U is negative and increases toward zero as they separate. A negative energy is allowed: it compares the configuration with the chosen zero, rather than describing a negative amount of motion.

Count each pair once. Three charges have three pairs; four charges have six. The potential energy is not assigned to a lone charge independently of the other charges. To assemble a configuration very slowly, an external agent does work equal to the change in potential energy. If kinetic energy also changes, external work is not simply ΔU .

Worked example: an equilateral triangle

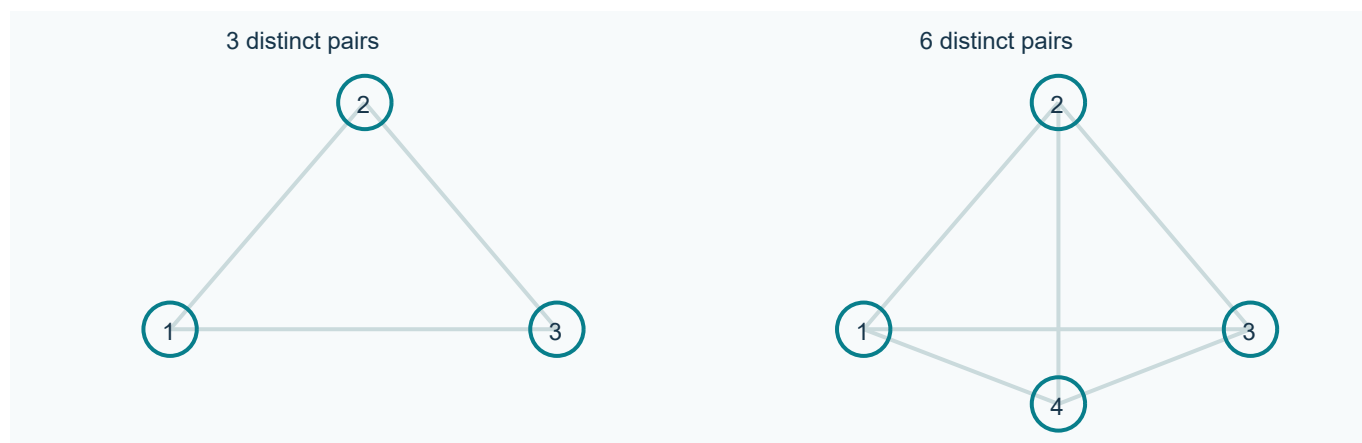
At the vertices of an equilateral triangle of side $a = 0.20$ m are $+q$, $+q$ and $-q$, with $q = 1.0$ μC . The three pair energies sum to $kq^2/a - kq^2/a - kq^2/a$. Thus **$U = -0.04495$ J**. The sign says the assembled configuration has lower energy than separated particles.

Move slowly	Sign of ΔU	External work
Separate opposite charges	Positive	Positive: an external agent supplies energy.
Bring like charges closer	Positive	Positive: work against repulsion.
Let like charges separate	Negative	Electric force can increase kinetic energy.

Teaching prompt

Ask students to sketch U versus separation for like and opposite charges. Then predict the force direction by asking which direction lowers U . Use the graph qualitatively; a calculus derivative is not required.

Pair counting and the gravitational analogy



Three charges have three interaction pairs. Adding a fourth charge adds three new pairs, for six pairs in total.

RELATIONSHIP AND CONDITIONS

$$U_{\text{electric}} = \sum kq_i q_j / r_{ij}$$

$$U_{\text{gravity}} = -Gm_1 m_2 / r$$

Both use zero interaction energy at infinite separation.

Comparison	Electric interaction	Gravitational interaction
Sign of force	Attractive or repulsive	Attractive for positive masses
Pair energy	Positive for like signs, negative for opposite signs	Negative for a bound pair with this zero
Adding sources	Potential and potential energy are scalars	Potential and potential energy are scalars
Force or field	Add vectors	Add vectors

In the elementary point-particle model, a single isolated particle has no pair interaction energy with another particle. This is the intended meaning of the original shorthand “a single charge has $U = 0$.” It does not mean that a charge has no rest energy or no possible self-energy in other models.

PAUSE AND EXPLAIN

The electric and gravitational force magnitudes each vary as $1/r^2$. Their ratio for two fixed masses and charges does not depend on separation. Explain why neutral macroscopic matter can still be dominated by gravity.

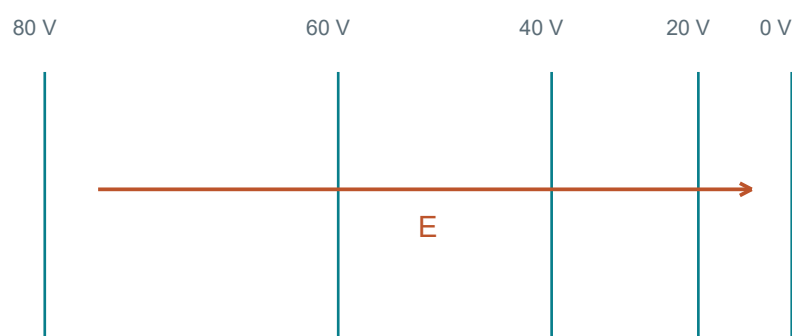
Potential, voltage and equipotentials

RELATIONSHIP AND CONDITIONS

$$V = kQ/r; V_{\text{net}} = \sum kQ_i/r_i; \Delta U = q\Delta V$$

Uniform field component: $E_x = -\Delta V/\Delta x$. Unit of potential: $1 \text{ V} = 1 \text{ J/C}$.

Potential is scalar, so source potentials add algebraically without x- and y-components. Potential difference is the change in potential energy per unit charge moved. A voltmeter compares two points; the zero of potential is a choice. Changing that zero does not change force, field or any physically measurable energy difference.



Equal voltage steps: closer lines mean stronger field.

The field points toward lower potential. For equal potential steps, tighter spacing means a larger field magnitude.

An equipotential has the same V everywhere. Moving a charge along it gives $\Delta U = 0$ and zero work by the electrostatic force. The field is perpendicular to the equipotential. A positive charge released from rest initially accelerates toward lower V ; a negative charge initially accelerates toward higher V . For arbitrary initial velocity, the instantaneous motion need not follow either direction.

Worked example: a potential gradient

Along x , V falls from 80 V to 20 V over 0.030 m in a uniform region. $E_x = -(20 - 80)/0.030 = \mathbf{+2000 \text{ V/m}}$.
Moving $q = -2.0 \text{ } \mu\text{C}$ through this change gives $\Delta U = (-2.0 \times 10^{-6})(-60) = \mathbf{+1.20 \times 10^{-4} \text{ J}}$.

On a V -versus- x graph, a horizontal segment means $E_x = 0$, and a steeper negative slope means a larger positive E_x . The finite-difference equation describes an average component; it is not a universal magnitude formula for an arbitrary curved path.

Electric work: which force and which system?

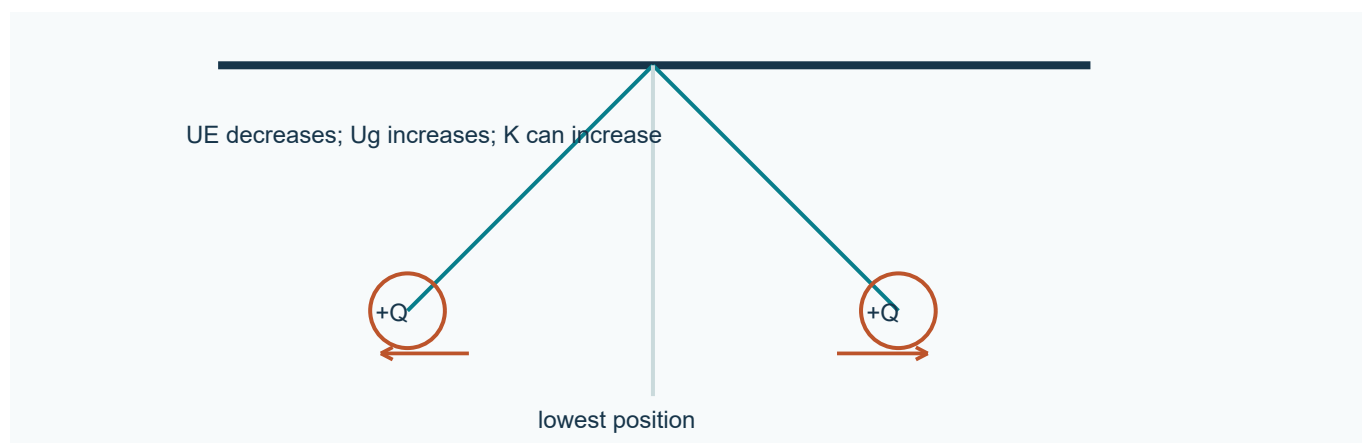
RELATIONSHIP AND CONDITIONS

$$W_{\text{electric}} = -\Delta U = -q\Delta V$$

$$W_{\text{external}} = \Delta K + \Delta U \text{ when the electric interaction is included in the system}$$

$$\text{Slow controlled motion: } \Delta K \approx 0, \text{ so } W_{\text{external}} \approx \Delta U$$

The original notes compare two choices of system. If Earth belongs to the system, use gravitational potential energy. If Earth lies outside the chosen system, account for the work of gravity instead. The same logic applies to electric interactions: include the potential energy once, or count the external interaction as work.



The source's like-charged spheres move apart and rise when released. A system containing both spheres and Earth includes both electric and gravitational potential energies.

Original example structure: slowly bring two +Q spheres from very far apart to separation r at the same height. With negligible change in kinetic and gravitational energies, $W_{\text{external}} = kQ^2/r$. After release, for the two-sphere-plus-Earth system, $\Delta K + \Delta U_E + \Delta U_g = 0$.

For a system containing only the two spheres, gravity is external. As the spheres rise, it does negative work and the sum $K + U_E$ decreases. The two descriptions predict the same motion.

PAUSE AND EXPLAIN

Why does “external work equals ΔU ” require the slow-motion or unchanged-kinetic-energy condition? If you also speed the particle up, some supplied work becomes kinetic energy.

Conductors, spheres and shielding

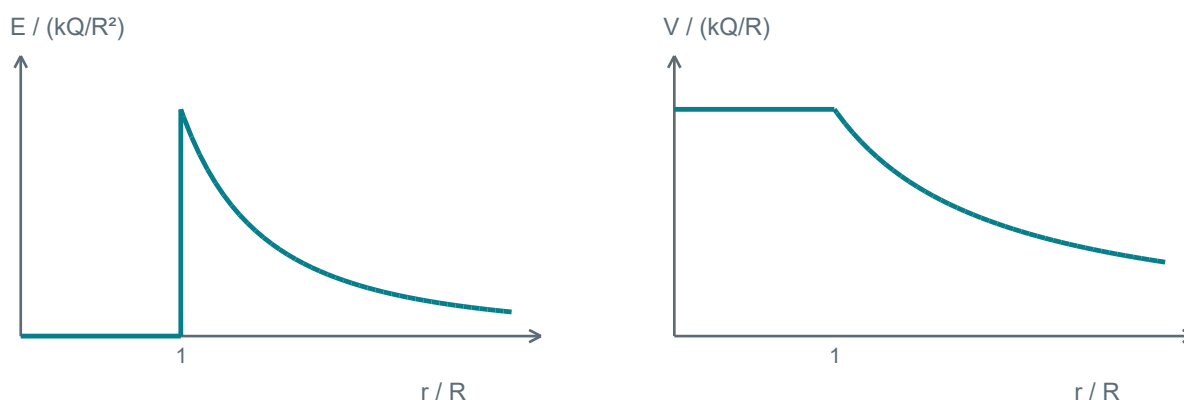
In electrostatic equilibrium, the electric field inside conducting material is zero. Otherwise its mobile charges would continue to move. Excess charge lies on its surface, and the field just outside is perpendicular to that surface. The entire connected conductor is at one potential; this potential need not be zero.

RELATIONSHIP AND CONDITIONS

Isolated conducting sphere of radius R , charge Q ; choose $V = 0$ at infinity:

outside: $E = k|Q|/r^2$, $V = kQ/r$

inside: $E = 0$, $V = kQ/R$ (constant)



Graphs for $Q > 0$. Potential is continuous at the surface; the idealized electric field changes discontinuously across surface charge.

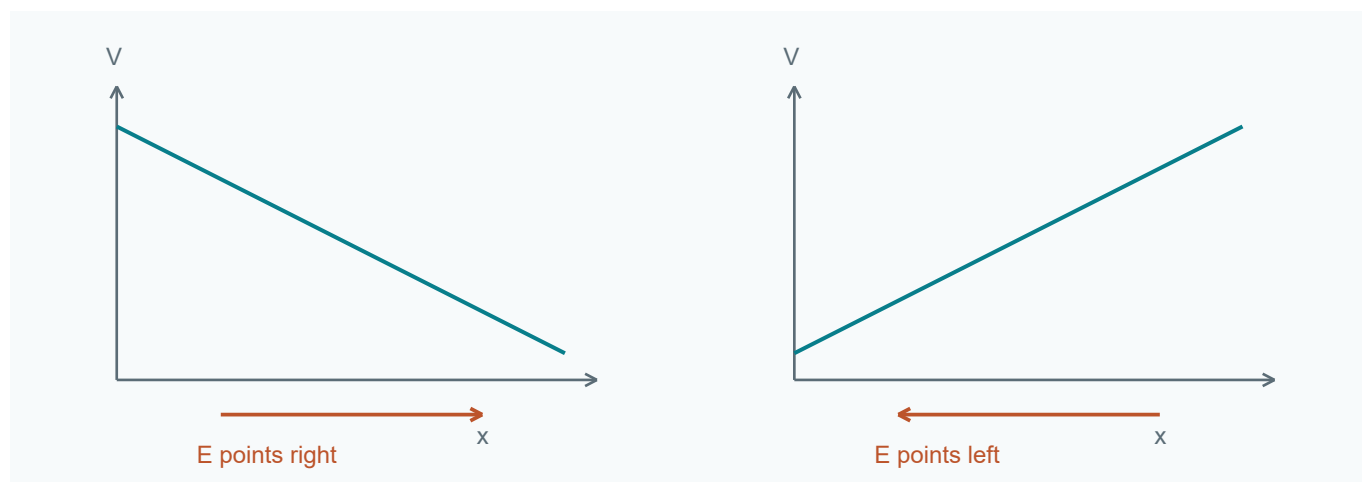
The outside point-charge model requires spherical symmetry. A nearby charged object can distort the surface charge distribution and invalidate that symmetry. An insulator can retain charge inside its volume and have a nonzero internal field. Do not extend the conductor result to every charged object.

Worked example

A metal sphere has $R = 0.10$ m and $Q = +4.0$ nC. At its center, $\mathbf{E} = \mathbf{0}$ but $V = 360$ V. At $r = 0.20$ m, $\mathbf{E} = 899$ N/C outward and $V = 180$ V. Moving a charge inside the same conductor changes no electric potential energy.

An empty closed cavity inside a conductor in electrostatic equilibrium is shielded from an external static field. If a charge is placed inside the cavity, charges are induced on the cavity wall; the field inside the conducting material still vanishes, but the cavity field need not vanish. This qualification prevents overusing the phrase “inside a conductor.”

Field and potential: the graph connection



In one dimension, the sign of the field is opposite the slope of $V(x)$. Flat potential means zero field component.

RELATIONSHIP AND CONDITIONS

Uniform field component: $E_x = -\Delta V/\Delta x$

General local statement: $E_x = -dV/dx$ (slope interpretation)

$$W_E = q(V_i - V_f)$$

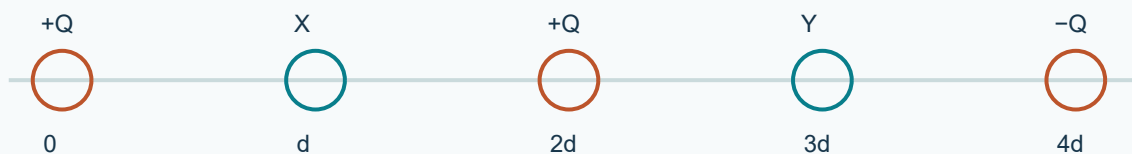
The original derivation connects force, work and potential. For a small displacement along x , electric work is $qE_x \Delta x$ and also $-q\Delta V$. Cancel q to connect the field component with the negative potential slope. The derivative notation explains the local idea but AP work can use slopes and finite intervals.

$V(x)$ feature	E_x	Force on a negative charge
Decreases linearly	Positive and constant	Toward negative x
Increases linearly	Negative and constant	Toward positive x
Horizontal	Zero	No x -component of electric force
Steeper slope magnitude	Larger $ E_x $	Larger force magnitude for the same q

EQUIPOTENTIAL SPACING

For equal voltage steps, close contours mean a larger field. The electric field is perpendicular to the contour and points toward lower V . The sign of the test charge changes its force, not the field map.

The original three-charge line: E and V rank differently



The original collinear configuration: the positions X and Y are between neighboring source charges.

Place $+Q$ at $x = 0$, $+Q$ at $x = 2d$ and $-Q$ at $x = 4d$. Point X is at d and point Y is at $3d$. Include all three source charges when calculating each field or potential.

Contribution	At X	At Y
Field from left $+Q$	$+kQ/d^2$	$+kQ/(9d^2)$
Field from middle $+Q$	$-kQ/d^2$	$+kQ/d^2$
Field from right $-Q$	$+kQ/(9d^2)$	$+kQ/d^2$
Net field	$kQ/(9d^2)$, right	$19kQ/(9d^2)$, right

RELATIONSHIP AND CONDITIONS

$$V_X = kQ/d + kQ/d - kQ/(3d) = 5kQ/(3d)$$

$$V_Y = kQ/(3d) + kQ/d - kQ/d = kQ/(3d)$$

Thus $E_Y > E_X$, but $V_X > V_Y$. A proton brought slowly from infinity requires more external work to reach X: $W_X = 5keQ/(3d)$, while $W_Y = keQ/(3d)$. Both are positive.

WHY THIS EXAMPLE MATTERS

Field uses signed directional components and inverse-square distance. Potential uses source-charge signs and inverse distance. A location with the larger field need not have the larger potential. Dropping the farther charge gives an incorrect potential even when it is a small field contribution.

Materials, gravity and the charge model

Gravity between positive masses is attractive; electric force can attract or repel. For two objects that carry fixed charges, $F_E/F_g = k|q_1q_2|/(Gm_1m_2)$: the separation cancels. Electric forces often dominate between charged particles. Gravity dominates many astronomical systems because large bodies are nearly electrically neutral while their masses add.

In a conductor, mobile charge carriers can redistribute throughout the connected material. In an insulator, charges are much less mobile, but positive and negative charge distributions can shift slightly relative to each other: **polarization**. A charged rod can therefore attract a neutral object. Attraction alone does not prove opposite net charges; repulsion is stronger evidence of like net signs.

RELATIONSHIP AND CONDITIONS

$Q_{\text{system,final}} = Q_{\text{system,initial}}$ if no charge crosses the boundary.

Permittivity ϵ describes a material's electric response; $\epsilon = \kappa\epsilon_0$ for a simple linear dielectric.

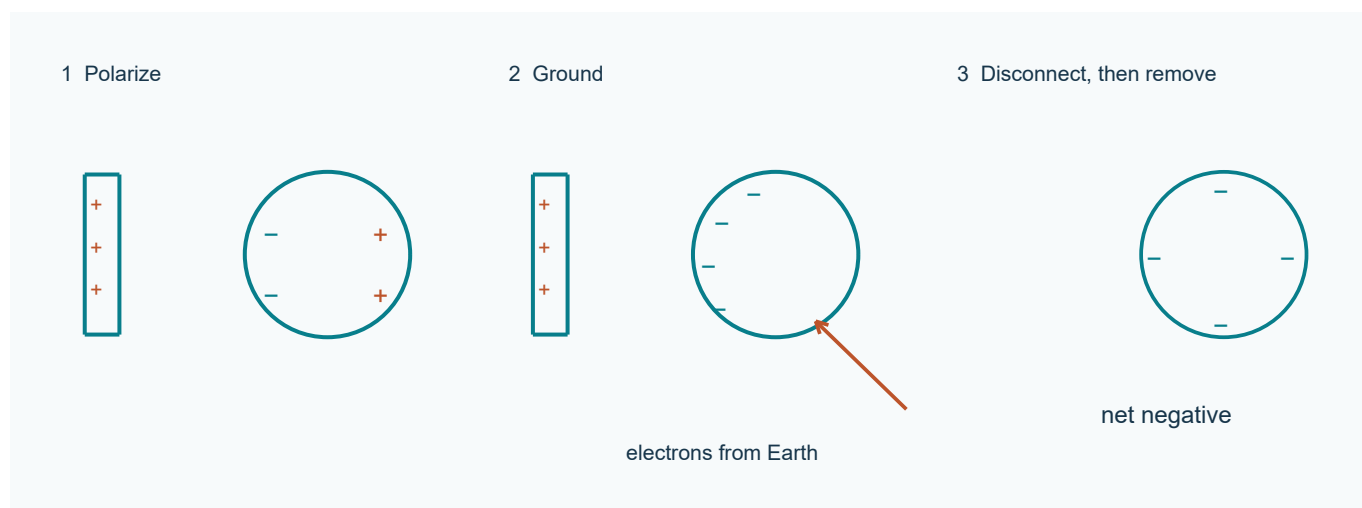
Process	What physically happens	Net charge?
Friction	Electrons transfer between unlike materials.	Each object can change; total stays fixed.
Contact	Charges redistribute through electrical contact.	Connected objects share charge.
Polarization	Charges redistribute within an object.	No net change is required.
Grounding	Electrons transfer to or from a large reservoir.	The object is no longer charge-isolated.

Reason through induction

Bring a negative rod near a neutral conductor. Electrons shift away from the rod. Ground the conductor while the rod remains: some electrons leave. **Disconnect ground first**, then remove the rod. The conductor remains positively charged. Removing the rod while grounding remains allows it to return toward neutrality.

For two identical isolated conducting spheres, contact followed by separation gives each half the total charge, by symmetry. For unequal conductors, equal potential is the equilibrium requirement; equal charge is not generally correct. Microscopic electric interactions underlie many everyday contact forces such as normal force and friction.

Charging by contact and induction



A positive external rod draws electrons toward a conductor. Grounding can supply extra electrons. Disconnect the ground before removing the rod to leave negative net charge.

Contact

Electrons transfer until connected conductors reach the same potential. Identical isolated spheres sharing a total charge split that charge equally. Different sizes generally share potential rather than equal charge. For widely separated spheres, equal potential gives $Q_1/R_1 = Q_2/R_2$.

Induction with grounding

Bring the positive rod close without touching. Connect the conductor to ground while the rod remains nearby. Electrons enter from Earth. Remove the ground first, then remove the rod. The excess negative charge spreads over the conductor. Removing the rod before disconnecting the ground can let the excess charge return.

POLARIZATION IS DIFFERENT FROM NET CHARGING

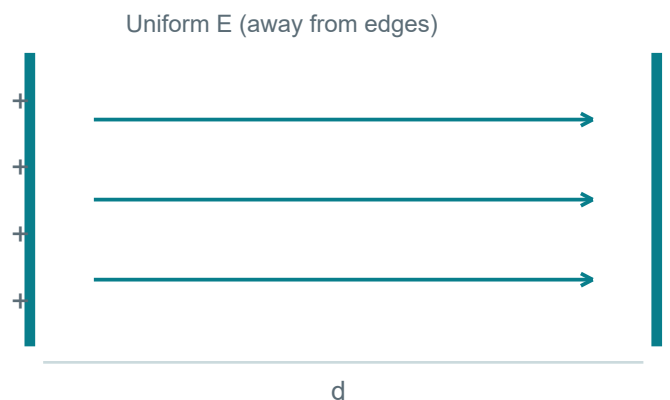
A nearby rod can separate charge without changing net charge. A neutral conductor can be attracted because its nearer opposite charge feels the stronger force. Attraction alone does not prove that the object carries opposite net charge.

Capacitance and the parallel-plate model

RELATIONSHIP AND CONDITIONS

$$C = Q/|\Delta V|; C = \kappa\epsilon_0 A/d; |E| \approx |\Delta V|/d$$

Q is the magnitude on either plate; the two plates carry $+Q$ and $-Q$. C is in farads (C/V).



A parallel-plate capacitor separates charge and stores field energy. The uniform-field approximation applies away from the edges when d is small compared with plate dimensions.

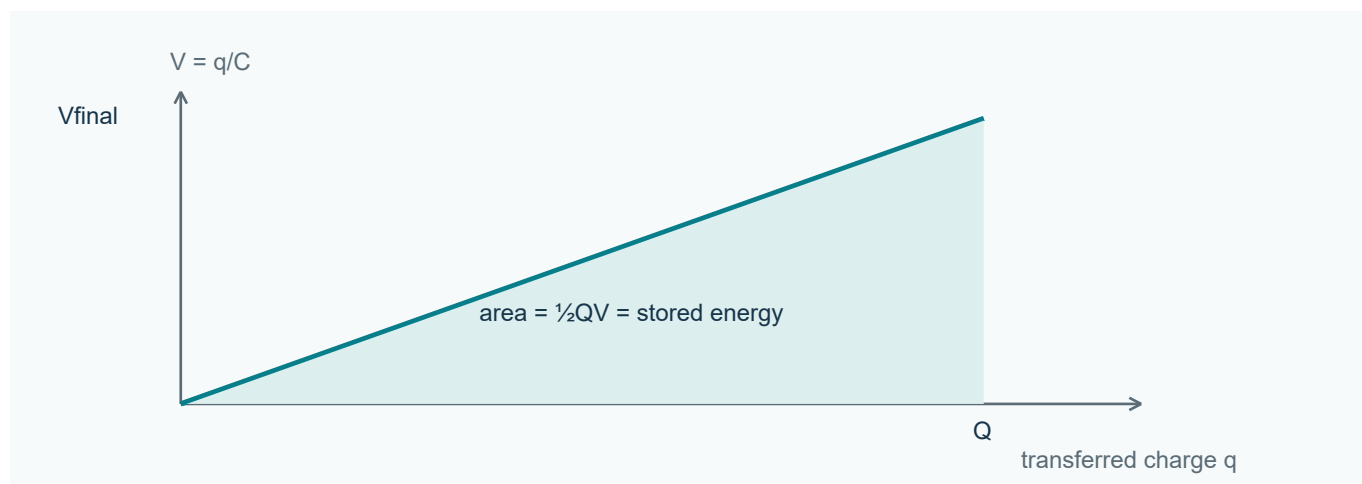
Capacitance is determined by geometry and dielectric properties in the ideal linear model, not by independently choosing Q or V . Increasing plate area provides more room for separated charge; increasing separation makes a given charge separation produce a larger voltage, so capacitance falls. A dielectric polarizes and produces a field opposing the field of the free plate charges.

Worked example

Air plates have area 0.020 m^2 and separation 1.0 mm . $C = (8.85 \times 10^{-12})(0.020)/(1.0 \times 10^{-3}) = 1.77 \times 10^{-10} \text{ F} = 177 \text{ pF}$. Connected to 100 V , they carry $Q = 17.7 \text{ nC}$ and produce approximately $E = 1.0 \times 10^5 \text{ V/m}$.

The total net charge of the usual two-plate capacitor is zero. "Charge on a capacitor" conventionally means the magnitude on one plate. Charge does not cross the ideal insulating gap during ordinary charging; electrons move through the external circuit onto one plate and off the other.

Why capacitor energy contains a factor of one half



As the capacitor charges from 0 to Q , $V = q/C$ rises linearly. The area under V versus transferred charge is a triangle.

RELATIONSHIP AND CONDITIONS

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

A battery or other external agent first transfers charge against a small potential difference. Later charge is moved against a larger potential difference. The average voltage during charging from zero is $V/2$, so the total energy stored is $Q(V/2)$.

The original plate-force argument: the field E between ideal oppositely charged plates is twice the field produced by either plate alone. The force on one plate comes from the other plate, so its magnitude is $Q(E/2)$. This explains the same factor of one half in the source sketch. A plate does not push itself with its own field.

EXPLICIT CORRECTION

The final formula is $Q^2/(2C)$. Omitting the 2 doubles the predicted energy. Use either the triangular charging graph or $U = \frac{1}{2}QV$ to check the result.

Example: $C = 4.0 \mu\text{F}$ and $V = 10 \text{ V}$ give $Q = 40 \mu\text{C}$ and $U = 200 \mu\text{J}$, not $400 \mu\text{J}$.

Capacitor energy and dielectric changes

RELATIONSHIP AND CONDITIONS

$$U_C = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

The factor $\frac{1}{2}$ arises because the voltage rises from zero as an initially uncharged capacitor is charged.

Insert dielectric, $\kappa > 1$	Connected to ideal battery	Disconnected, isolated plates
Capacitance C	Multiplies by κ	Multiplies by κ
Voltage $ \Delta V $	Fixed by battery	Divides by κ
Plate charge Q	Multiplies by κ	Fixed by isolation
Field E (fixed d)	Unchanged	Divides by κ
Stored energy U	Multiplies by κ	Divides by κ

These are different physical systems. With the battery connected, it moves additional charge and supplies energy. With the capacitor isolated, no plate charge can enter or leave. The reduction in field energy can become mechanical work as the dielectric is pulled into the gap, or other energy if motion is dissipative. Stored field energy alone is not the conserved energy of the full apparatus.

Worked comparison

A $2.0 \mu\text{F}$ capacitor initially has 12 V across it: $Q = 24 \mu\text{C}$ and $U = 144 \mu\text{J}$. Insert $\kappa = 3$ fully. **Battery connected:** $C = 6.0 \mu\text{F}$, $Q = 72 \mu\text{C}$, $V = 12 \text{ V}$, $U = 432 \mu\text{J}$. **Disconnected:** $C = 6.0 \mu\text{F}$, $Q = 24 \mu\text{C}$, $V = 4.0 \text{ V}$, $U = 48 \mu\text{J}$.

Common trap

Using $U = \frac{1}{2}CV^2$ to say “larger C always stores more energy” silently assumes fixed V . At fixed Q , $U = \frac{Q^2}{2C}$, so larger C stores less energy.

Doubling plate separation gives $C/2$. Connected to a fixed battery, Q halves and E halves. Disconnected with fixed Q , V doubles while E remains unchanged in the ideal parallel-plate approximation. Use this as a second check of the same reasoning.

Electric work and particle motion

RELATIONSHIP AND CONDITIONS

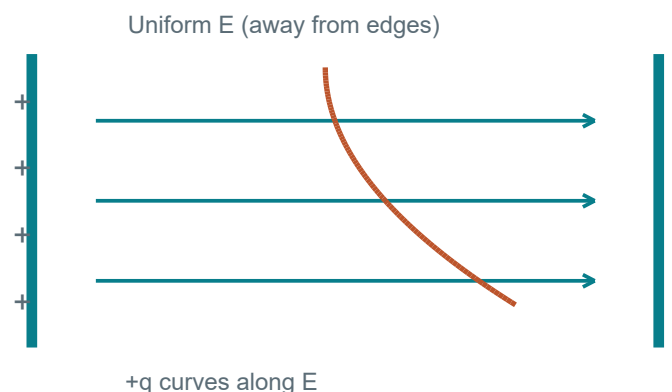
$$W_{\text{electric}} = -\Delta U = -q\Delta V$$

If only the electric force does work: $\Delta K = -q\Delta V$, so $K_i + U_i = K_f + U_f$.

Choose the system before writing conservation of energy. If both interacting charges move, include both kinetic energies. If one source is fixed by a support, only the free particle changes kinetic energy in the simple model. A magnetic field alone cannot provide the kinetic-energy increase that an electric potential drop can.

Worked example: accelerating an electron

An electron starts from rest and moves through a potential increase of +200 V. $\Delta U = (-e)(+200 \text{ V})$, so $K_f = 200 \text{ eV} = 3.20 \times 10^{-17} \text{ J}$. With $m_e = 9.11 \times 10^{-31} \text{ kg}$, $v = \sqrt{2K/m} = 8.38 \times 10^6 \text{ m/s}$. The negative charge is why an increase in V lowers U.



A positive particle launched downward between these plates bends to the right. Electric force produces a constant horizontal acceleration in the uniform-field model.

Resolve motion into a component parallel to E and a component perpendicular to E . The parallel acceleration is qE/m ; the perpendicular velocity is constant if no other forces act. Find the transit time using the direction with known constant velocity, then use the accelerated-motion equations in the other direction. Energy gives the speed change; component kinematics gives the trajectory.

An experiment that links V, E and C

A | Map electric potential

Use a low-voltage supply, conducting paper and two fixed electrodes. Keep a reference probe on one electrode. Measure potential at many marked positions with a second probe. Locate several points at each selected voltage and draw smooth equipotential curves. Draw local field arrows perpendicular to those curves, pointing from higher to lower potential. Estimate a component using $-\Delta V/\Delta x$ for closely spaced points.

Keep electrode geometry and supply voltage fixed. Repeat readings, avoid dragging the reference contact, and use equal voltage intervals when comparing line spacing. Probe spacing that is too large averages over a changing field. Contact resistance and irregular paper conductivity can distort the map; compare symmetric regions to detect these effects.

B | Determine an unknown capacitance

Charge the capacitor at several measured voltages and measure the magnitude of transferred charge using suitable charge-measurement equipment. Plot Q vertically against ΔV horizontally. In the linear regime, the slope is C, in $C/V = F$. A nonzero intercept can indicate a charge offset or residual initial charge. Discharge between runs and hold plate geometry and dielectric fixed.

Analysis with sample data

At 2.0, 4.0, 6.0 and 8.0 V, measured charges are 3.0, 6.1, 9.0 and 12.1 μC . An endpoint estimate gives slope $(12.1 - 3.0)/(8.0 - 2.0) = \mathbf{1.52 \mu F}$. A best-fit line using all points is better than one Q/V ratio. Compare residual scatter with the instrument resolution.

A clear experimental answer specifies the measured variables, the controlled variables, repeated trials, the graph and its slope, and a physically explained source of uncertainty. "Human error" is not a mechanism.

Student practice

1. Identical spheres carry $+6.0\text{ nC}$ and -2.0 nC . They touch and separate. Find each final charge and the number of electrons transferred.
2. Two $+3.0\text{ nC}$ charges are 0.40 m apart. Find E and V at their midpoint, with $V = 0$ at infinity.
3. A $+2.0\text{ }\mu\text{C}$ charge moves from 50 V to -20 V . Find ΔU , electric work and the change in kinetic energy if no other force does work.
4. Three charges $+2.0\text{ nC}$, $+2.0\text{ nC}$ and -2.0 nC form an equilateral triangle of side 0.10 m . Find total potential energy.
5. A $4.0\text{ }\mu\text{F}$ capacitor is charged to 10 V and then disconnected. A dielectric with $\kappa = 2$ is inserted fully. Find C , Q , V and U after insertion.

Chapter review: potentials and motion

6. The same insertion occurs while the capacitor remains on the 10 V battery. Find the final energy. Explain why this differs from question 5.
7. At $x = 0, 0.020$ and 0.040 m, $V = 90, 60$ and 30 V. Find E_x . Which way does an electron accelerate?
8. A proton starts at rest and gains 800 eV in an electric field. Find its speed using $m_p = 1.67 \times 10^{-27}$ kg. State the sign of its potential change.
9. A conductor is charged and in electrostatic equilibrium. A student says “ $E = 0$ inside, so $V = 0$ inside.” Explain the error.
10. A $+1.0$ nC charge is placed in a uniform field 3.0×10^3 N/C directed right. Its mass is 2.0×10^{-6} kg. Ignore gravity and air resistance. If its initial velocity is downward at 0.50 m/s, find its rightward displacement after it descends 0.10 m.

Practice solutions and reasoning

1. Total charge $+4.0 \text{ nC}$, so each ends at $+2.0 \text{ nC}$. The first sphere gains 4.0 nC worth of electrons from the second: $N = 4.0 \times 10^{-9} \text{ e} = \mathbf{2.50 \times 10^{10} \text{ electrons}}$.
2. Equal opposite field vectors cancel: $\mathbf{E = 0}$. Potentials add: $V = 2k(3.0 \times 10^{-9})/0.20 = \mathbf{270 \text{ V}}$.
3. $\Delta V = -70 \text{ V}$. $\Delta U = q\Delta V = \mathbf{-1.40 \times 10^{-4} \text{ J}}$; $W_{\text{E}} = \Delta K = \mathbf{+1.40 \times 10^{-4} \text{ J}}$.
4. The signed pair sum is $-kq^2/a = \mathbf{-3.60 \times 10^{-7} \text{ J}}$; there are three pairs, not six.
5. $C = \mathbf{8.0 \text{ }\mu\text{F}}$, $Q = \mathbf{40 \text{ }\mu\text{C}}$ remains fixed, $V = Q/C = \mathbf{5.0 \text{ V}}$, $U = \frac{1}{2}CV^2 = \mathbf{100 \text{ }\mu\text{J}}$.

Worked answers: potentials and motion

6. Voltage remains 10 V, so $U = \frac{1}{2}(8.0 \mu\text{F})(10 \text{ V})^2 = \mathbf{400 \mu\text{J}}$. The battery supplies additional charge and energy.

7. $E_x = -(-30)/0.020 = \mathbf{+1500 \text{ V/m}}$. An electron accelerates left, toward higher potential.

8. $K = 800(1.60 \times 10^{-19}) \text{ J}$; $v = \sqrt{(2K/m_p)} = \mathbf{3.92 \times 10^5 \text{ m/s}}$. A positive charge gains energy by moving through $\Delta V = \mathbf{-800 \text{ V}}$.

9. Zero field requires constant potential, not a particular numerical value. Grounding or a chosen reference can set a value; it is not implied by $E = 0$.

10. $t = 0.10/0.50 = 0.20 \text{ s}$. $a_x = qE/m = 1.5 \text{ m/s}^2$. $x = \frac{1}{2}a_x t^2 = \mathbf{0.030 \text{ m right}}$. The downward component is unaccelerated in the stated model.

Original notes and syllabus map

The source sequence determines the teaching order. Repeated examples are consolidated while distinct reasoning, diagrams and comparisons are retained.

Original source	Content retained
N01, pp.1-4 Electrostatics 10.1, 10.3-10.5.pdf	Force, field, signed pair energy and potential
N02, pp.1-3 Electrostatics 10.3-10.5, 10.7 + Circuits 11.1-11.3, 11.5.pdf	Electric work, conducting sphere and the field-potential slope
N04, pp.6-9 Circuits 11.2-11.7 + Electrostatics 10.6.pdf	Capacitance, the factor $1/2$ and dielectrics
N05, pp.1-2 Electrostatics 10.6 + Circuits 11.2-11.8.pdf	Capacitor energy and charge accounting
N27, pp.1-3,7-10 Problem Solving - Electrostatics 10.1-10.5, 10.7 + Magnetism 12.2 + Modern Physics 15.7-15.8.pdf	System choices, charged pendulums and polarization
N35, pp.5-12 Problem Solving - Thermal 9.1-9.2, 9.4 + Electrostatics 10.1-10.5, 10.7.pdf	Induction, contours, three-charge line and perpendicular vectors

Curriculum: supplied *AP Physics 2 Course and Exam Description*, 2026, Unit 10. Topic labels on each content page show where the syllabus connects to the original lesson. Extensions remain labeled.

Clarifications and independent references

Point checked	Treatment in this chapter
Energy factor	The capacitor formula $Q^2/(2C)$ includes 1/2.
Three-charge line	All three sources contribute to both V and E .
Work and system	External work equals ΔU only when other relevant energy changes vanish.
Conductor	$E = 0$ implies constant V , not necessarily $V = 0$.
Dielectric	Specify connected battery (fixed V) or isolated plates (fixed Q).

Independent checks

[OpenStax: electric potential and field](#)

[OpenStax: energy stored in a capacitor](#)

[OpenStax: molecular model of a dielectric](#)

Constants and calculations use the values stated in each example. Historical practice provides context and reasoning patterns. New worked questions and corrected explanations are distinguished from an official answer key.

Master summary: force, field, energy and potential

Quantity	Definition / point-charge result	How contributions combine	What it predicts
Force F	$F = qE$; magnitude $k qQ /r^2$	Vector sum	Acceleration of the chosen test charge
Field E	Field of point source: kQ/r^2 radially, with sign	Vector sum	Force per unit positive test charge
Pair energy U	$U = kqQ/r$	Scalar sum over distinct pairs	Energy of a configuration; zero chosen at infinity
Potential V	$V = kQ/r$; $\Delta V = \Delta U/q$	Scalar sum over sources	Energy per charge and electrostatic work
Capacitance C	$C = Q/ \Delta V = \kappa\epsilon_0 A/d$	Geometry and dielectric set C	Charge needed for a specified voltage
Stored capacitor energy	$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/(2C)$	Compare under stated constraint	Energy in the electric field

Change	Battery connected	Isolated plates
Insert dielectric κ	$C \times \kappa$, $Q \times \kappa$, V fixed, $U \times \kappa$	$C \times \kappa$, Q fixed, V/κ , U/κ
Double separation d	$C/2$, $Q/2$, V fixed, $E/2$	$C/2$, Q fixed, $V \times 2$, E unchanged

Work: $W_E = -q\Delta V$. With electric force alone, $\Delta K = -q\Delta V$. Electric field points toward lower potential; the force on a negative charge points opposite the field.