

# Geometric Optics

## Teaching and student notes

Reflection and refraction lead into one shared way of understanding lenses and mirrors. Learn the converging pair together, then the diverging pair. Use the image-side rule to translate between the two instruments.

### THE CENTRAL CONNECTION

A convex lens and a concave mirror have the same image type, orientation, magnification and signed image-distance relation for the same real-object distance and positive focal length. The physical image side differs because a lens transmits light and a mirror reflects it.

| Route                               | Main idea                                                                       | Pages |
|-------------------------------------|---------------------------------------------------------------------------------|-------|
| 1. Light at a boundary              | Reflection, the road-to-sand analogy, Snell's law and total internal reflection | 2-8   |
| 2. The converging pair              | Convex lens + concave mirror: draw, compare and explain each image case         | 9-15  |
| 3. Equations and the diverging pair | Signs, magnification, original numerical examples and comparisons               | 16-18 |
| 4. Understanding and checking       | Half-covered lens, material comparison, experiments and practice                | 19-24 |
| 5. Reference                        | Source map and the large image summary table                                    | 25-26 |

## How to use these notes

Before calculating, predict the image type and location. Draw two rays from the same object point. Then use the equation and check whether the signs agree with the sketch. Each “Pause and explain” prompt can be used as a short class discussion.

**Scope:** CED topics 13.1-13.4. Unless stated otherwise, objects are real, lenses are thin, mirrors are spherical, rays are near the principal axis, and the surrounding medium is air. “Same side” always means the same physical side as the object.

**Notation:**  $s_o$  = object distance;  $s_i$  = image distance;  $f$  = focal length;  $m$  = signed transverse magnification. Positive  $s_i$  means real; negative  $s_i$  means virtual. Frequency is written  $f_{\text{wave}}$  where it might be confused with focal length.

# Reflection and the plane mirror

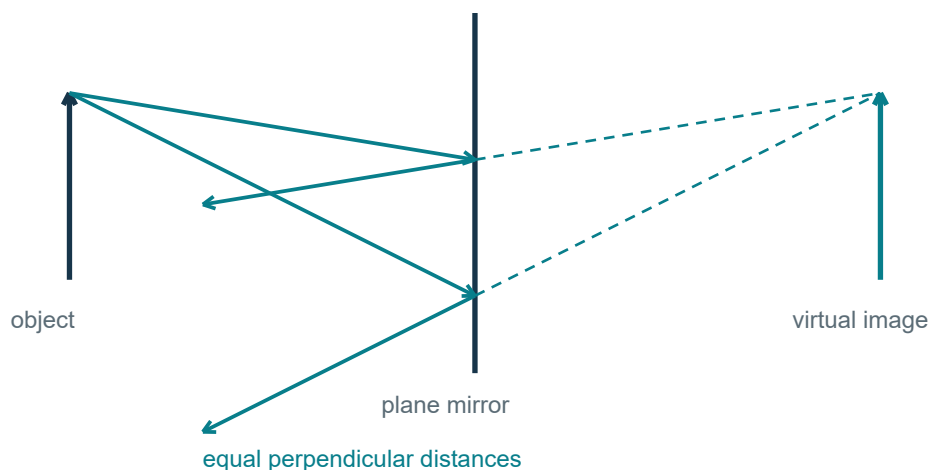
A ray shows the direction in which light travels. It is perpendicular to a wavefront. In a uniform medium it is straight. Ray optics works when wave spreading, interference and diffraction can be neglected; those effects need the wave model in Unit 14.

## The normal is the reference line

Draw a line perpendicular to the surface where the ray strikes. The incident and reflected angles are measured from that normal, not from the surface. The reflected ray remains in the incident medium.

### LAW OF REFLECTION

$\theta_i = \theta_r$ . If a ray makes  $25^\circ$  with a mirror surface, both angles to the normal are  $65^\circ$ .



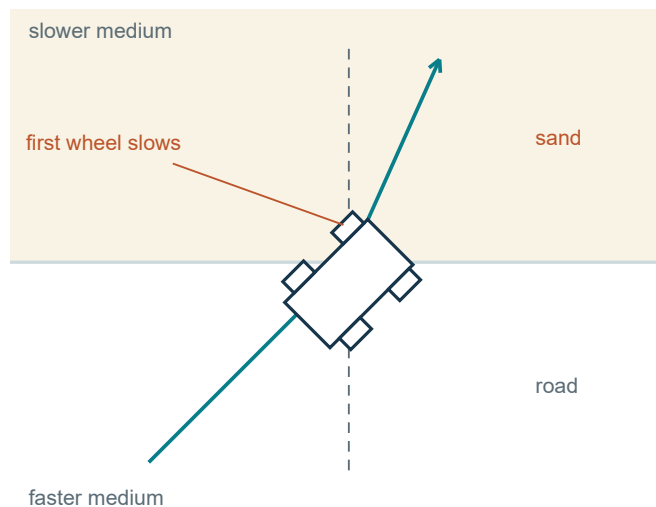
The reflected rays diverge. Extend them backward to find where they appear to originate. The image is **virtual, upright and the same size**, at the same perpendicular distance behind the mirror as the object is in front. A screen behind the mirror cannot collect rays that never go there.

| Smooth surface                                                                                         | Rough surface                                                                                             |
|--------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| Specular reflection: nearby surface normals are nearly parallel, so a parallel beam remains organized. | Diffuse reflection: local normals point in different directions. Each ray still obeys the reflection law. |

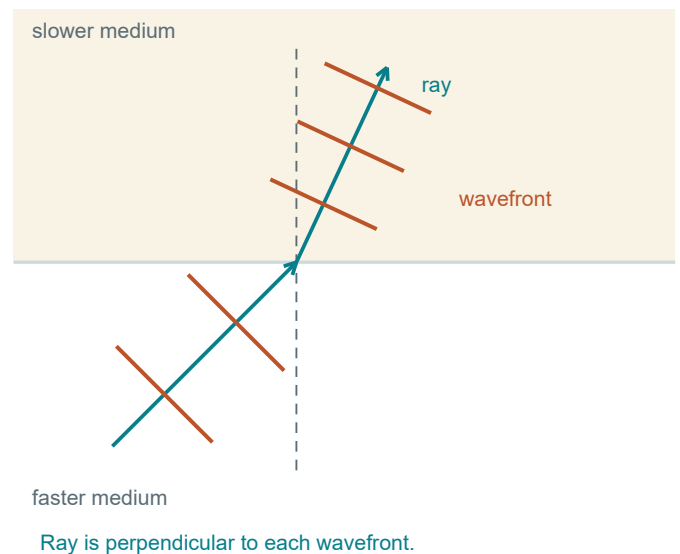
**Pause and explain:** Why does a rough wall scatter light without breaking the reflection law? If the object moves 3 cm toward a plane mirror, the image moves 3 cm toward the mirror from behind; their separation falls by 6 cm.

# Refraction: the road-to-sand analogy

## Car: road to sand



## Wavefronts: unequal advance



**Road-to-sand model, redrawn from the original notes:** a small car crosses obliquely from a fast concrete road into slower sand. The first wheel to enter the sand slows first; the other wheel continues faster for a moment. The car turns toward the normal. Leaving sand reverses the process and turns it away from the normal.

## Transfer the idea to a wavefront

A wavefront is a line of points at the same phase, such as a crest. When one end reaches the slower medium first, that part advances a shorter distance while the other end is still moving faster. The wavefront rotates. Because rays are perpendicular to wavefronts, the ray changes direction too.

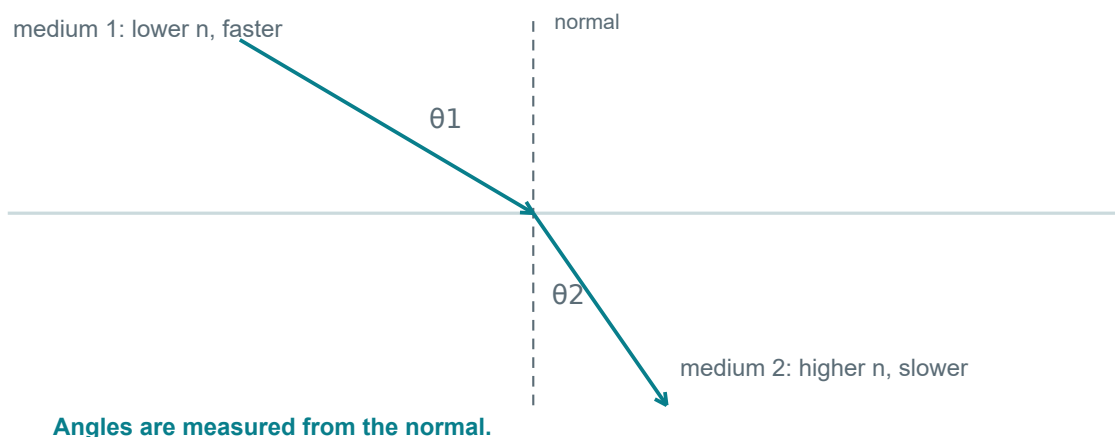
### WHAT CHANGES AT A STATIONARY BOUNDARY?

Speed changes because the medium changes. Frequency stays the same because the boundary is driven at the source frequency. Since  $v = f_{\text{wave}} \lambda$ , wavelength changes in the same ratio as speed.

**Limits of the analogy:** the wheels explain unequal advance and turning. Light is not literally a car, and refraction does not require frictional energy loss. A transparent medium can change speed without absorbing the wave's energy.

**Pause and explain:** At normal incidence, both wheels enter together. No turning occurs. Similarly, a normally incident ray remains straight, even though its speed and wavelength can change.

# Snell's law: turn the picture into numbers



## INDEX, SPEED AND ANGLE

$n = c/v$ , so a larger refractive index means a lower light speed.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_1 / \sin \theta_2 = v_1/v_2 = n_2/n_1$$

| Direction of travel        | Speed and wavelength                 | Ray direction                        |
|----------------------------|--------------------------------------|--------------------------------------|
| Lower $n$ to higher $n$    | Both decrease; frequency stays fixed | Toward the normal                    |
| Higher $n$ to lower $n$    | Both increase; frequency stays fixed | Away from the normal, if transmitted |
| Incidence along the normal | May change with the medium           | No bending                           |

## Worked example: air into glass

A 600 nm vacuum-wavelength beam enters glass of  $n = 1.50$  at  $30^\circ$  to the normal. With  $n_{\text{air}} \approx 1.00$ :

**$\sin \theta_{\text{glass}} = \sin 30^\circ / 1.50 = 1/3$ , so  $\theta_{\text{glass}} = 19.5^\circ$ .**

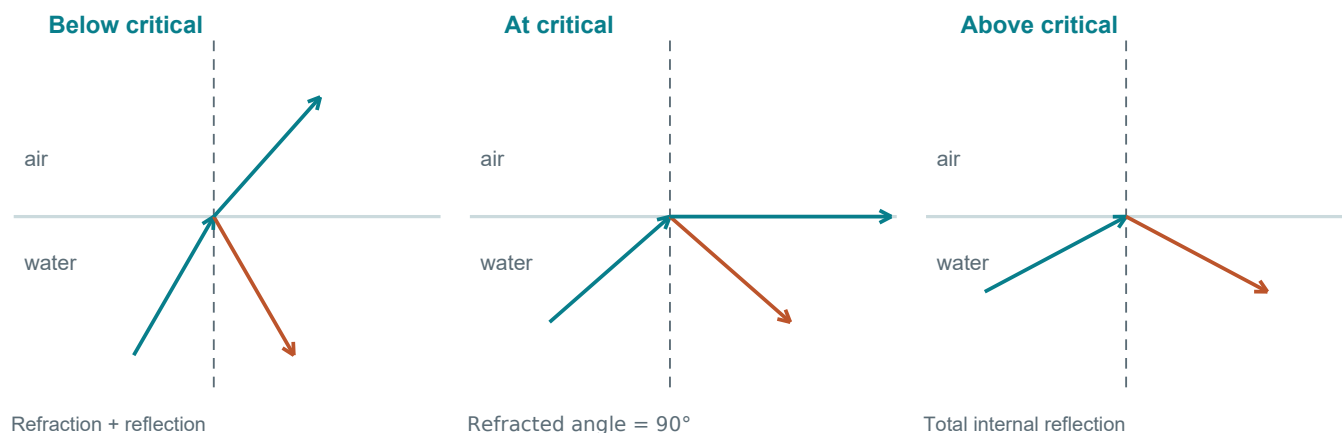
$v = c/n = 2.00 \times 10^8 \text{ m/s}$ ;  $\lambda_{\text{glass}} = 600/1.50 = 400 \text{ nm}$ .

Frequency stays  $c/(600 \text{ nm}) = 5.00 \times 10^{14} \text{ Hz}$ .

**Checks:** the glass angle is smaller; the speed and wavelength are lower. At  $\theta_1 = 0^\circ$ , Snell's law gives  $\theta_2 = 0^\circ$ . "No refraction" in a sketch means no change in direction, not unchanged speed.

# Critical angle and total internal reflection

Start with light traveling from higher  $n$  to lower  $n$ . The transmitted ray bends away from the normal. Increase the incident angle: the transmitted angle grows until it reaches  $90^\circ$ . The corresponding incident angle is the **critical angle**.



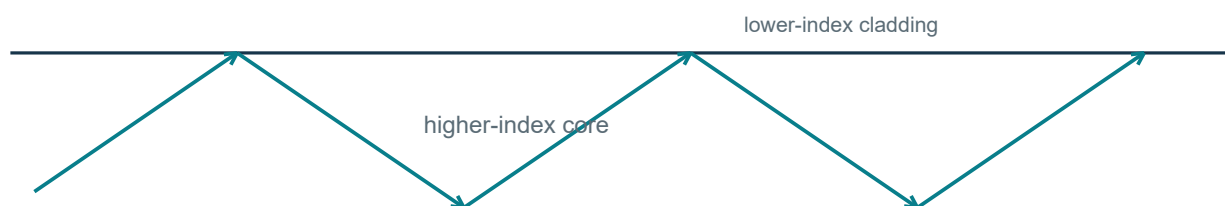
## TWO CONDITIONS, BOTH REQUIRED

1. Light travels from  $n_1$  to  $n_2$ , with  $n_1 > n_2$ .
2.  $\theta_i > \theta_c$ , where  $\sin \theta_c = n_2/n_1$ .

At  $\theta_i = \theta_c$ , the limiting refracted ray lies along the surface. Beyond it, there is no propagating transmitted ray in the elementary ray model.

**Derivation:** put  $\theta_2 = 90^\circ$  into Snell's law:  $n_1 \sin \theta_c = n_2$ . The ratio must be less than 1. Air-to-water light therefore cannot show this type of total internal reflection.

## Optical fibre: keeping the light inside



The fibre core has a larger refractive index than its surroundings or cladding. Rays meeting the side at a sufficiently large angle to the **local normal** are internally reflected. Do not confuse that angle with the angle to the fibre axis.

**Precision:** "optically denser" means higher refractive index here, not necessarily higher mass density. A sharply bent fibre may let some rays fail the critical-angle condition.

# Refraction practice from the original notes

## Example A: infer $n$ , then find the critical angle

Light goes from air into a transparent material. Its angle changes from  $30.0^\circ$  to  $23.0^\circ$ , both measured from the normal. Find the material-to-air critical angle.

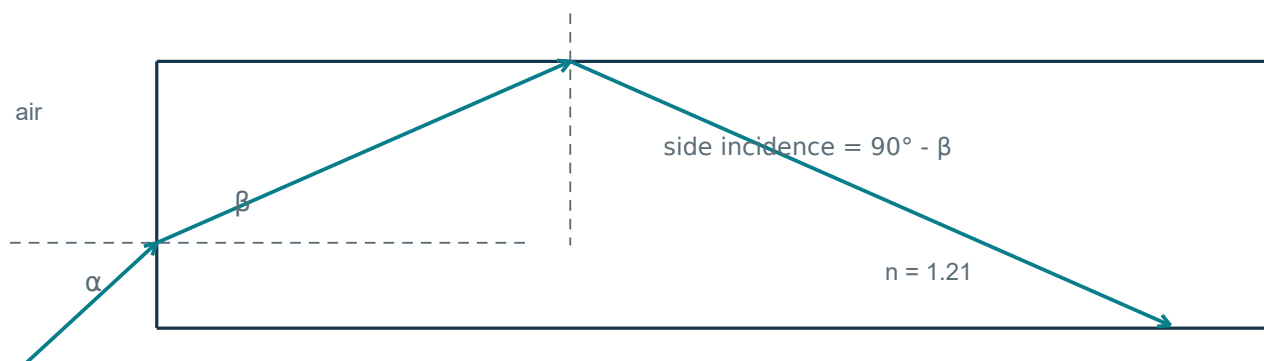
**Step 1:**  $n = \sin 30.0^\circ / \sin 23.0^\circ = 1.280$ .

**Step 2:**  $\sin \theta_c = 1/n = \sin 23.0^\circ / \sin 30.0^\circ$ .

**Answer:**  $\theta_c = 51.4^\circ$ . The critical angle belongs to a ray going *out of* the material, not the original air-to-material ray.

## Example B: the largest entrance angle for a fibre

A straight fibre has  $n = 1.21$  and is surrounded by air. Light enters its flat end from air. Let  $\alpha$  be its external angle to the fibre axis, and  $\beta$  its internal angle to the axis. What is the largest  $\alpha$  that still allows total internal reflection at the side?



At the side,  $\theta_c = \sin^{-1}(1/1.21) = 55.7^\circ$ . The limiting  $\beta$  is  $90^\circ - \theta_c = 34.3^\circ$ .

At the entrance,  $\sin \alpha = 1.21 \sin \beta$ . Therefore  $\alpha_{\max} = 42.9^\circ$  (the critical-angle limit).

### WHY THIS IS A TWO-BOUNDARY PROBLEM

The entrance face and the side wall have different normals. Apply Snell's law at the entrance and the total-internal-reflection condition at the wall. The angles  $\beta$  and the wall incidence angle are complementary.

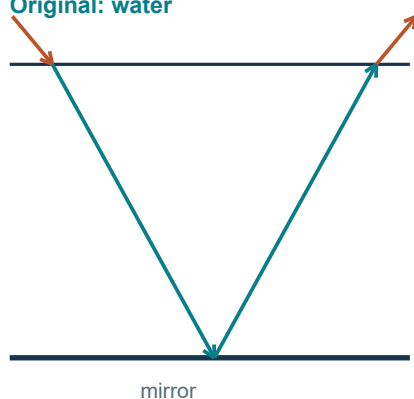
**Pause and explain:** Why does entering the fibre successfully not guarantee that the light stays trapped? Sketch a ray with too large an entrance angle.

# Layered media and a mirrored tank

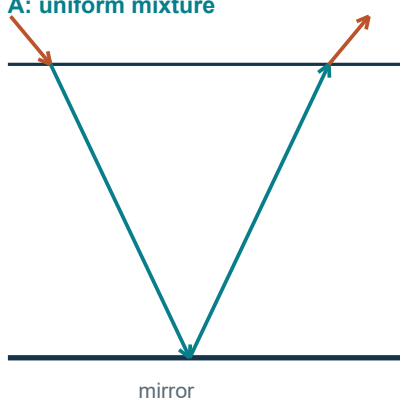
The original practice compares a tank of water, a uniformly mixed higher-index liquid, and three horizontal layers. Preserve the reasoning chain: **draw the path** → **track frequency and wavelength** → **rank angles** → **compare horizontal travel**.

| Tank     | Refractive index structure                                                |
|----------|---------------------------------------------------------------------------|
| Original | Water, $n_w$                                                              |
| Model A  | Uniform mixture, $n_m > n_w$                                              |
| Model B  | Three equal-depth layers: $n_w, n_m, n_b$ , where $n_b > n_m > n_w > n_a$ |

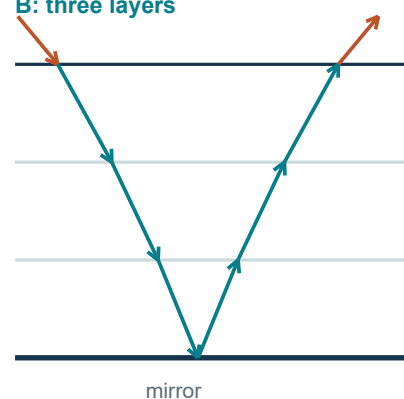
Original: water



A: uniform mixture



B: three layers



**Across parallel interfaces:**  $n \sin \theta$  stays constant, so intermediate indices cancel. In the bottom layer,  $\sin \theta_4 = (n_a/n_b) \sin \theta_1$ . In Model A the angle is  $\theta_1$ ; in Model B the top, middle and bottom angles are  $\theta_2, \theta_3, \theta_4$ . Thus  $\theta_2 > \theta_1 = \theta_3 > \theta_4$ .

**Wavelength:** entering a higher-index liquid reduces speed and wavelength; frequency is unchanged. Reflection at the horizontal bottom reverses the vertical direction but preserves the angle to the normal. On returning to air, the exit angle equals the entry angle.

## HORIZONTAL-DISTANCE ARGUMENT

A down-and-up path through depth  $h$  contributes  $2h \tan \theta$ . Both modified tanks spend at least part of the path at an angle smaller than in pure water, and no part at a larger angle. Therefore  $d_A < d_w$  and  $d_B < d_w$ . No comparison between  $d_A$  and  $d_B$  is required.

# Refraction checks and useful extensions

## A compact reasoning checklist

| Question                             | Reasoning to use                                                                                        |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|
| Which way does the ray bend?         | Identify the two indices and draw the normal before comparing angles.                                   |
| What stays unchanged?                | At a stationary interface, frequency remains fixed. Speed and wavelength can change.                    |
| Can total internal reflection occur? | Check higher $n$ to lower $n$ first, then compare incidence to the critical angle.                      |
| Can a ray cross several layers?      | For parallel boundaries, use $n \sin \theta = \text{constant}$ . Draw a new normal if a boundary tilts. |
| Does a mirror change the index?      | No. Reflection returns the ray to the same medium.                                                      |

## Dispersion: different wavelengths can bend differently

The refractive index can depend on wavelength. In ordinary visible-light glass, violet generally has a higher index than red, so it bends more strongly. A prism can separate white light. This is an extension of the same Snell's-law reasoning, not a separate bending rule.

## Extension: apparent depth

A submerged object seen from above can appear closer to the surface because the eye extends the refracted rays backward. For viewing close to the normal through a flat surface:

$$\text{apparent depth} \approx \text{real depth} \times n_{\text{observer}} / n_{\text{object medium}}$$

A point 1.20 m below water with  $n = 1.33$  appears about 0.90 m deep to a near-normal observer in air. This simple expression is not a general formula for oblique viewing.

### PAUSE AND EXPLAIN

A ray enters a parallel-sided glass slab from air and leaves into air. Why is the emerging direction parallel to the original direction? Why can the emerging line still be displaced sideways?

**Reasoning:** Snell's law at the two parallel faces gives equal external angles. The ray spent part of its journey traveling in a different direction inside the slab, so its final line can be laterally shifted. At normal incidence there is neither bending nor lateral shift.

# Two pairs, one image-formation framework

| Shared behavior     | Lens         | Mirror         |
|---------------------|--------------|----------------|
| Converging: $f > 0$ | Convex lens  | Concave mirror |
| Diverging: $f < 0$  | Concave lens | Convex mirror  |

For lenses, convex/concave describe shape; converging/diverging describe behavior. The shape-to-behavior pairing above assumes the lens index is greater than that of its surroundings. For mirrors, the corresponding behavior comes from reflection at the curved surface.

## What makes an image real or virtual?

**Real:** outgoing rays from one object point actually meet. A screen placed there can intercept a focused image.

**Virtual:** outgoing rays spread apart, but their backward extensions meet. The eye receives the real outgoing light and locates its apparent origin. “Virtual” does not mean invisible.

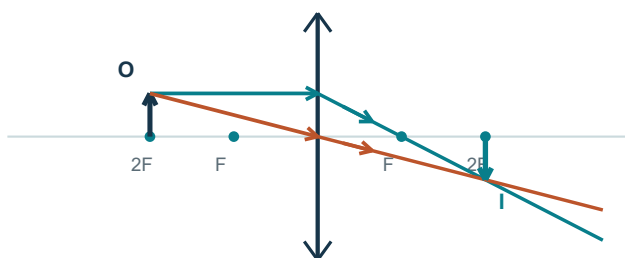
### THE IMAGE-SIDE RULE

Lens: real image on the opposite side from the object; virtual image on the same side.

Mirror: real image on the same side as the object; virtual image behind the mirror, on the opposite side.

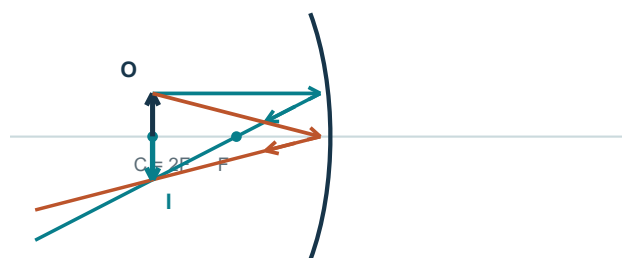
**Reason:** real images lie on the outgoing-light side. A lens transmits; a mirror sends light back.

### Convex lens



Solid: light path Dashed: backward extension

### Concave mirror



Solid: light path Dashed: backward extension

O labels the object and I the image. These diagrams use the same  $f$  and  $s_o$ . Both produce a real, inverted image of equal size. The image is  $2f$  from the instrument in each case, but its physical side changes. Lens symbols and shallow mirror sketches represent the thin/paraxial model.

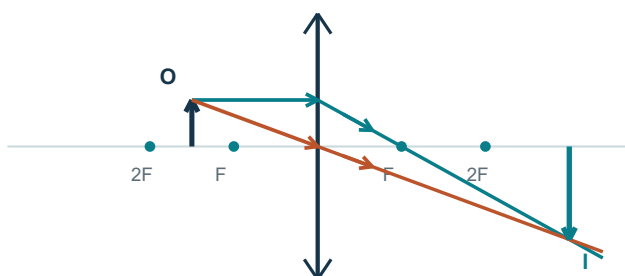
**Important limit:** “Only change the image side” is a shortcut for translating the *image-case results*. Lenses and mirrors still have different ray rules, physical mechanisms and focal geometry.

# Principal rays: compare the two instruments

Start every construction with rays from the **same object point**, usually the top of an upright arrow. Two rays determine the image point; a third checks the result. Label the principal axis, focal point(s), object and image.

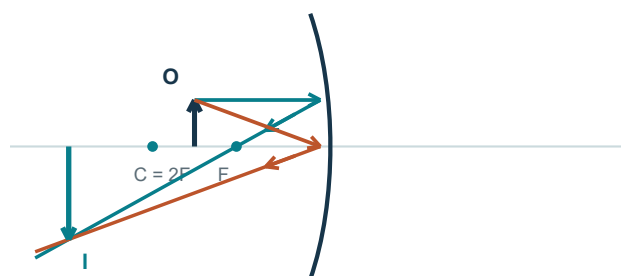
| Ray choice           | Convex lens                                                  | Concave mirror                                                |
|----------------------|--------------------------------------------------------------|---------------------------------------------------------------|
| Parallel to axis     | After transmission, passes through the far focal point       | After reflection, passes through the near focal point         |
| Through / toward F   | Through near F before the lens → parallel after transmission | Through F before the mirror → parallel after reflection       |
| Central / vertex ray | Through the optical center → approximately undeviated        | At the mirror vertex → reflects with equal angles to the axis |
| Extra mirror check   | Do not import this as a lens rule                            | Through center of curvature C → retraces its path             |

## Convex lens



Solid: light path Dashed: backward extension

## Concave mirror



Solid: light path Dashed: backward extension

## Focal length and curvature

A spherical mirror has  $f \approx R/2$  for paraxial rays, where  $R$  is its signed radius of curvature. For a concave mirror,  $F$  and  $C$  are on the object side; for a convex mirror they are behind the mirror. A plane mirror is the infinite-radius limit, with an infinitely distant focus.

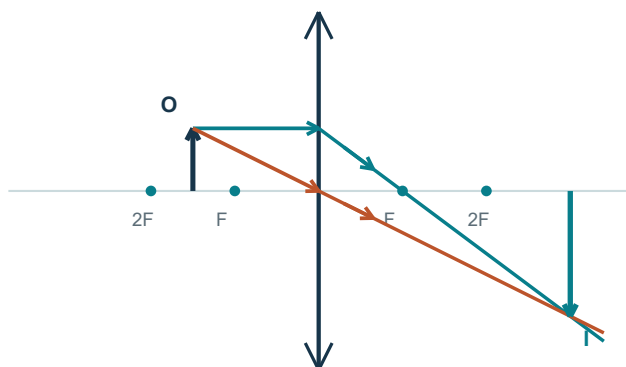
A thin lens in the same medium on both sides has a focal point on each side, at equal distances from the optical center. Lens focal length depends on refractive index and both surface curvatures; do not apply  $f = R/2$  to a lens.

**Pause and explain:** Why is the central lens ray drawn straight, while the ray hitting a mirror's vertex must turn back?

## Between $f$ and $2f$ : the projector case

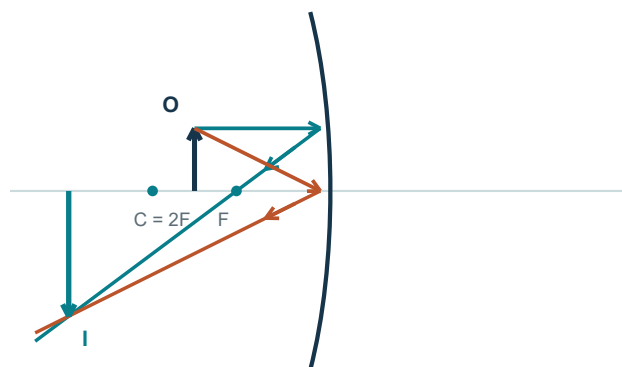
Begin with the case used in the original convex-lens notes: put a real object between the focal point and twice the focal length. Both instruments converge the rays beyond  $2f$ .

### Convex lens



Solid: light path Dashed: backward extension

### Concave mirror



Solid: light path Dashed: backward extension

| Property                  | Convex lens                | Concave mirror             |
|---------------------------|----------------------------|----------------------------|
| Object                    | $f < s_o < 2f$             | $f < s_o < 2f$             |
| Image distance            | $s_i > 2f$                 | $s_i > 2f$                 |
| Type / orientation / size | Real / inverted / enlarged | Real / inverted / enlarged |
| Physical image side       | Opposite side from object  | Same side as object        |

### A numerical pair

Use  $f = 10$  cm and  $s_o = 15$  cm for both instruments.

$1/s_i = 1/10 - 1/15 = 1/30$ , so  $s_i = +30$  cm.

$m = -s_i/s_o = -2$ . The image is twice as tall and inverted. Place it 30 cm beyond the lens, or 30 cm in front of the concave mirror.

#### APPLICATION: PROJECTOR

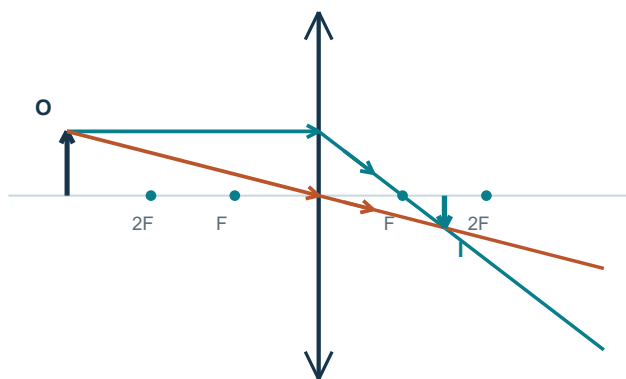
The convex lens forms an enlarged real image on a screen. A real image can be caught on a screen because the transmitted rays actually meet there. The slide or display orientation must account for inversion.

## Beyond 2f: the camera case

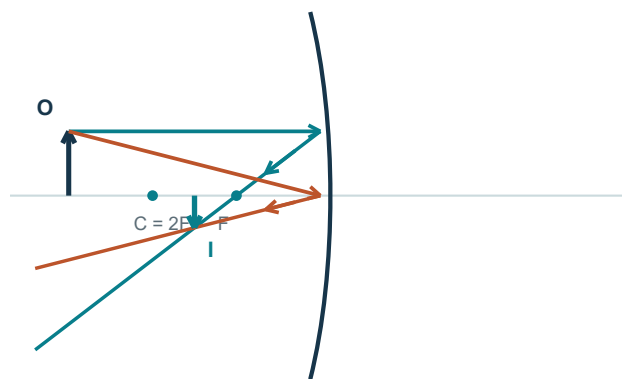
Move the object farther away than  $2f$ . The same construction now gives an image between  $f$  and  $2f$ . Apply the lens-to-mirror translation without relearning the image properties.

### Convex lens

### Concave mirror



Solid: light path Dashed: backward extension



Solid: light path Dashed: backward extension

| Property                  | Both converging instruments                      |
|---------------------------|--------------------------------------------------|
| Location from instrument  | $f < s_i < 2f$                                   |
| Type / orientation / size | Real / inverted / reduced                        |
| Physical side             | Lens: opposite side; mirror: same side as object |

### A numerical pair

For  $f = 10$  cm and  $s_o = 30$  cm:

$$1/s_i = 1/10 - 1/30 = 1/15, \text{ so } s_i = +15 \text{ cm.}$$

$m = -15/30 = -0.50$ . A 4 cm object gives a 2 cm inverted image.

#### APPLICATION: CAMERA

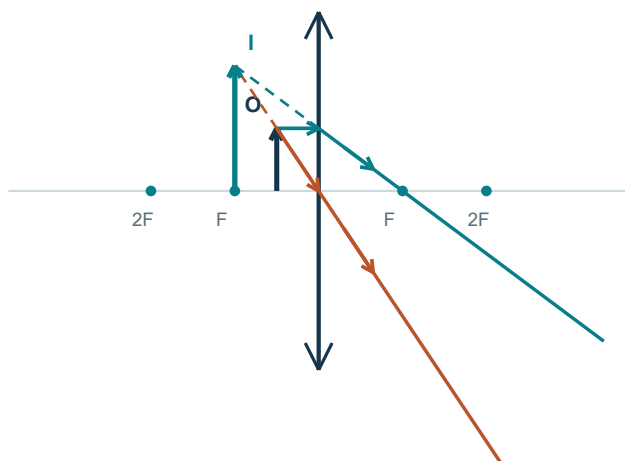
The camera lens produces a reduced real image of a distant scene on a film or sensor. As the object becomes very distant,  $s_i$  approaches  $f$ . Parallel rays from one distant object point focus in the focal plane; off-axis points focus away from the central focal point.

**Pause and explain:** A distant object moves closer, but stays outside  $2f$ . Does the image approach or move away from the lens? Does it get larger? Use both the ray picture and the equation.

# Inside f: the magnifier case

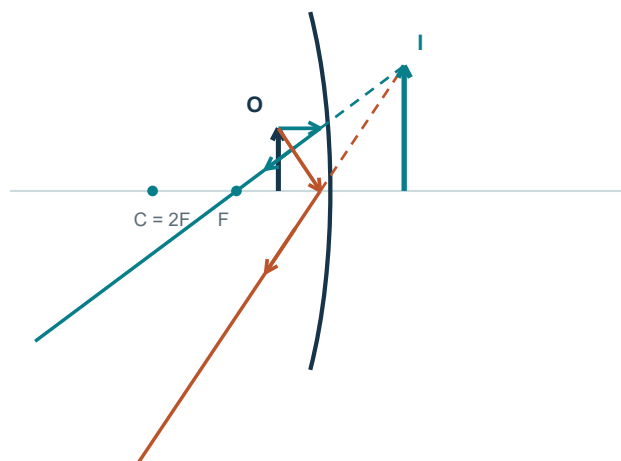
Place the object between the instrument and its focal point. The outgoing rays now diverge. Extend them backward: the extensions locate an upright enlarged virtual image.

## Convex lens



Solid: light path Dashed: backward extension

## Concave mirror



Solid: light path Dashed: backward extension

| Property            | Convex lens                | Concave mirror                   |
|---------------------|----------------------------|----------------------------------|
| Object position     | $0 < s_o < f$              | $0 < s_o < f$                    |
| Image               | Virtual, upright, enlarged | Virtual, upright, enlarged       |
| Physical image side | Same side as object        | Opposite side, behind mirror     |
| Typical use         | Magnifying glass           | Close-up shaving / makeup mirror |

For  $f = 10$  cm and  $s_o = 5$  cm:

$1/s_i = 1/10 - 1/5 = -1/10$ , so  $s_i = -10$  cm.

$m = -(-10)/5 = +2$ . Both give an upright image twice as tall. The negative image distance indicates virtuality, not an automatically fixed left/right side.

### WHY YOU CAN SEE A VIRTUAL IMAGE

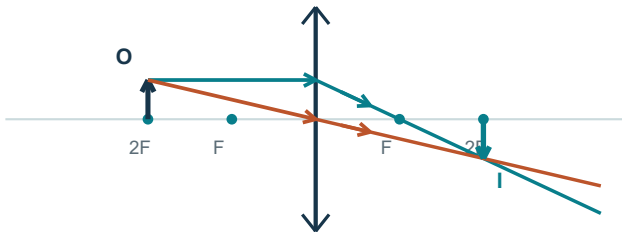
The eye receives actual diverging rays. Its own lens forms an image on the retina. The virtual image is the apparent source of those incoming rays; it cannot be caught directly on a screen placed at its apparent location.

**Pause and explain:** In the lens sketch the virtual image lies on the object side; in the mirror sketch it lies behind the mirror. Which parts of the sketch are extensions rather than paths actually traveled by light?

# The two boundary cases: $2f$ and $f$

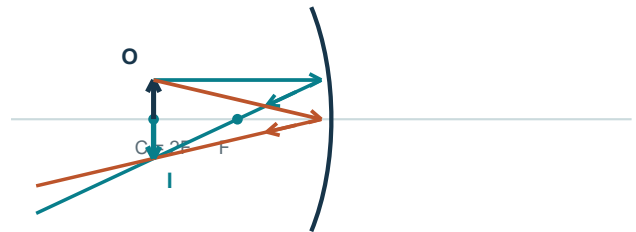
## Object at $2f$ : equal-size imaging

### Convex lens



Solid: light path Dashed: backward extension

### Concave mirror

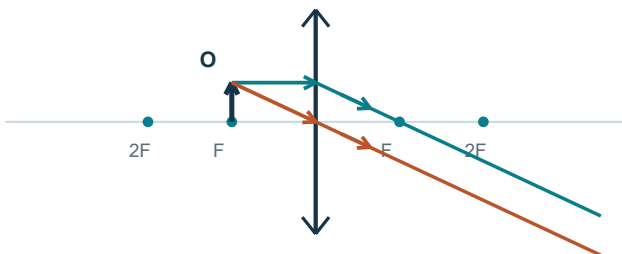


Solid: light path Dashed: backward extension

For either converging instrument,  $s_o = 2f$  gives  $s_i = 2f$  and  $m = -1$ . The image is real, inverted and the same size. The image-side rule still distinguishes the lens from the mirror.

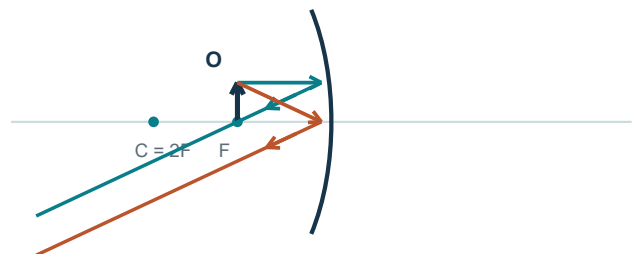
## Object at $f$ : parallel outgoing rays

### Convex lens



Solid: light path Dashed: backward extension

### Concave mirror



Solid: light path Dashed: backward extension

At  $s_o = f$ , the equation gives  $1/s_i = 0$ . The outgoing rays from an object point are parallel, so they do not meet at a finite distance. The ideal image is described as being **at infinity**.

### REFINING “NO IMAGE” IN THE ORIGINAL TABLE

Use “no finite image; outgoing rays parallel.” No finite screen position produces a focused image. This statement does not mean that light disappears or that an observer can receive no light.

**Approaching the boundary:** from outside  $f$ , the real image moves far away and becomes very large. From inside  $f$ , the virtual image is far away and enlarged. In both limits, the finite-size and paraxial approximations should be kept in mind.

# One continuous pattern, not five isolated facts

For a converging instrument, imagine moving the object toward it from far away. The familiar cases are landmarks along one continuous relationship.

| Object moves through...                  | What the image does                                                                                                  |
|------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Very far away $\rightarrow 2f$           | Real image moves from near $f$ to $2f$ and grows from small to equal size.                                           |
| $2f \rightarrow f$ , staying outside $f$ | Real image moves from $2f$ toward infinity and grows larger.                                                         |
| Exactly $f$                              | Outgoing rays are parallel; no finite image.                                                                         |
| Just inside $f \rightarrow$ very close   | Virtual image begins far away on the virtual side, then approaches the instrument; magnification falls toward $+1$ . |

## SAME ALGEBRA FOR BOTH

$$s_i = fs_o / (s_o - f)$$

$$m = -f / (s_o - f)$$

Outside  $f$ :  $s_i > 0$  and  $m < 0$ . Inside  $f$ :  $s_i < 0$  and  $m > 0$ .

## Explain the sign change

The numerator  $fs_o$  is positive for a real object and  $f > 0$ . Therefore the sign of  $s_i$  is controlled by the denominator  $s_o - f$ . This is why crossing the focal point changes the image from real to virtual. Magnification changes sign at the same boundary.

## Predict before drawing

1. A convex lens makes a real image three times as tall as the object. The object must be between  $f$  and  $2f$ , and the image is beyond  $2f$  on the opposite side.
2. Replace it with a concave mirror having the same  $f$  and the same object distance. The image is still real, inverted and three times as tall, but it is on the object side.
3. An upright enlarged image made by either instrument implies an object inside  $f$ , within the real-object model used here.

**Teaching check:** Students should be able to recreate the summary table from these relationships and the image-side rule, instead of memorizing two independent five-row tables.

# Signs, magnification and image description

## THE SHARED EQUATIONS

$$1/f = 1/s_o + 1/s_i$$

$$m = h_i/h_o = -s_i/s_o$$

| Quantity      | Positive                                 | Negative                                                  |
|---------------|------------------------------------------|-----------------------------------------------------------|
| $f$           | Converging: convex lens / concave mirror | Diverging: concave lens / convex mirror                   |
| $s_o$         | Real object, used throughout these notes | Virtual-object setups are outside this chapter's examples |
| $s_i$         | Real image                               | Virtual image                                             |
| $h_i$ and $m$ | Upright relative to upright object       | Inverted relative to upright object                       |

**Size comes from  $|m|$ :** greater than 1 means enlarged; equal to 1 means same size; less than 1 means reduced. A negative  $m$  does not mean “smaller.”

## Why $m = -s_i/s_o$

For a thin lens, use the ray through the optical center. The object and image triangles share an angle and have bases  $s_o$  and  $|s_i|$ . Their height ratio has magnitude  $|s_i|/s_o$ . A real image is below the axis for an upright object, producing the minus sign. The same signed relationship follows from the vertex-ray geometry for a paraxial mirror.

## Use signs before interpreting physical sides

Example: a result  $s_i = -18$  cm means a virtual image 18 cm from the instrument. For a lens it is on the object side; for a mirror it is behind the mirror. The numerical sign is tied to image type, while the instrument tells you the physical side.

## A COMPLETE ANSWER HAS FIVE PARTS

State the distance, physical side, real/virtual type, upright/inverted orientation, and enlarged/reduced/equal size. Example: “30 cm beyond the convex lens; real, inverted, twice the object height.”

**Common errors:** measuring object-to-image separation instead of distance from the instrument; using  $f > 0$  for every convex-shaped device; discarding a negative  $s_i$ ; reporting  $m$  without explaining its sign.

# The original paired numerical examples

## Example 1: an upright image twice as tall

A convex lens has  $f = +10$  cm and forms an upright image with  $m = +2$ . Find the real-object distance and image distance.

- 1. Use magnification first:**  $+2 = -s_i/s_o$ , so  $s_i = -2s_o$ .
- 2. Substitute:**  $1/10 = 1/s_o - 1/(2s_o) = 1/(2s_o)$ .
- 3. Solve:**  $s_o = 5$  cm;  $s_i = -10$  cm.
- 4. Interpret:** virtual, upright, twice as tall, 10 cm from the lens on the object side.

### TRANSLATE IT TO THE CONCAVE MIRROR

With the same  $f = +10$  cm and  $s_o = 5$  cm, the result remains  $s_i = -10$  cm and  $m = +2$ . Only the physical image side changes: 10 cm behind the mirror.

## Example 2: change to a diverging lens

Keep  $s_o = 5$  cm but use a concave lens with  $f = -10$  cm.

$$1/s_i = -1/10 - 1/5 = -3/10.$$

$$s_i = -10/3 \text{ cm} = -3.33 \text{ cm}.$$

$$m = -(-10/3)/5 = +2/3.$$

The image is virtual, upright and reduced, between the lens and its near focal point.

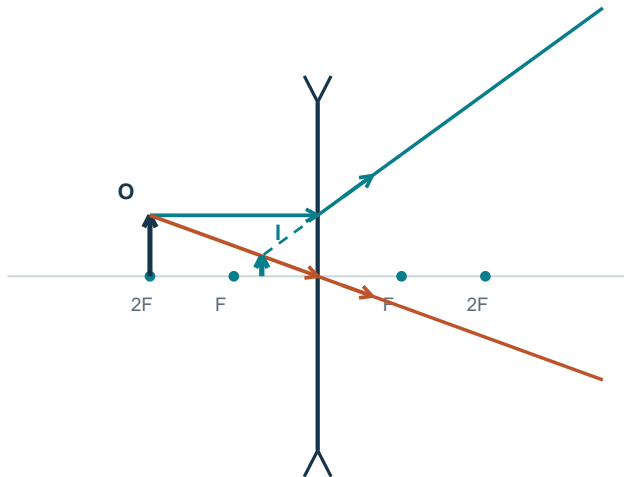
| Same real object distance: 5 cm | $f$    | $s_i$    | $m$  |
|---------------------------------|--------|----------|------|
| Convex lens / concave mirror    | +10 cm | -10 cm   | +2   |
| Concave lens / convex mirror    | -10 cm | -3.33 cm | +2/3 |

**Pause and explain:** Both calculations give virtual images. Why is one enlarged and the other reduced? The sign of  $s_i$  identifies virtuality; the ratio  $|s_i|/s_o$  determines the size.

# Concave lens + convex mirror: the diverging pair

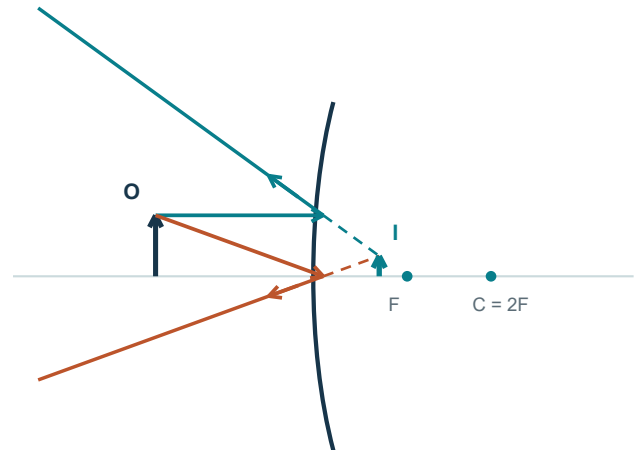
For a real object, these two instruments always give an **upright, reduced, virtual image**. A parallel incident ray leaves as if it came from the virtual focal point. Locate the image using backward extensions.

## Concave lens



Solid: light path Dashed: backward extension

## Convex mirror



Solid: light path Dashed: backward extension

| Property                       | Concave lens                                 | Convex mirror                                                     |
|--------------------------------|----------------------------------------------|-------------------------------------------------------------------|
| Focal length                   | Negative                                     | Negative                                                          |
| Image distance / magnification | $s_i < 0$ ; $0 < m < 1$                      | $s_i < 0$ ; $0 < m < 1$                                           |
| Image location                 | Same side as object, between lens and near F | Opposite side from object, behind mirror and between vertex and F |

## Prove the result rather than memorize it

Write  $f = -F$ , with  $F > 0$  and  $s_o > 0$ .

$s_i = -Fs_o/(s_o + F)$ , so it is always negative.

$m = F/(s_o + F)$ , so  $0 < m < 1$ .

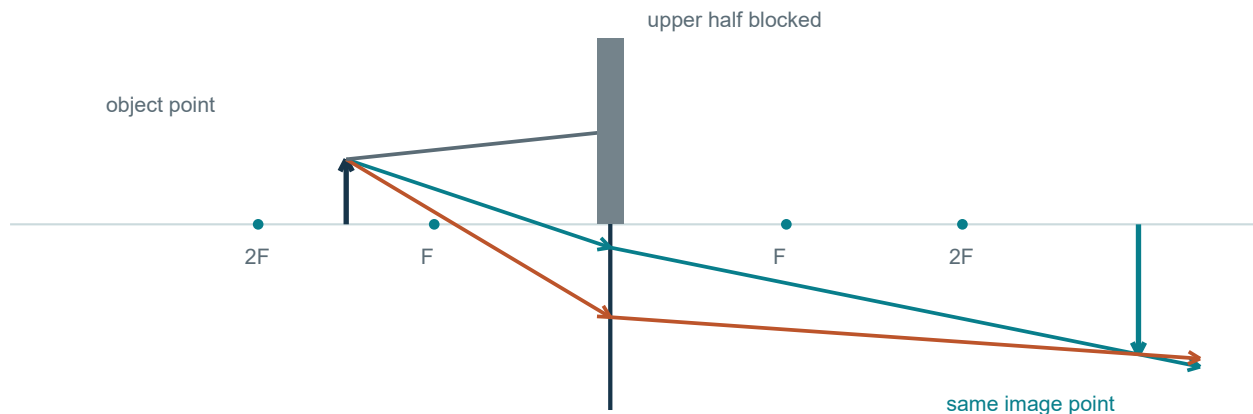
Also  $|s_i| = Fs_o/(s_o + F) < F$ : the image lies closer to the instrument than the virtual focus.

**Ray-rule distinction:** the central ray of a thin concave lens continues straight; a vertex ray at a convex mirror reflects. For a concave lens, a ray aimed toward the far focal point emerges parallel. For a convex mirror, a ray aimed toward the focus behind the mirror reflects parallel.

# Half a lens, and lenses made of different materials

## Covering half the lens does not remove half the image

The original drawing sends several rays from the same object point through different parts of the lens. Each object point normally illuminates a range of positions on the lens. Blocking some positions removes some rays from every image point, rather than deleting a particular half of the object.



Unblocked rays from this point still meet at the same image point.

### PREDICTION

In the ideal geometric model, the full image remains at the same location and with the same size, but it becomes dimmer. The focal length and the lens equation do not change. Aperture edges can introduce diffraction, which belongs to the wave model.

## Same shape, different refractive indices

The original comparison uses three lenses with the same size and shape but made of different materials. In the same surrounding medium, a larger lens refractive index gives stronger refraction and, for these converging lenses, a shorter focal length.

**How to compare ray diagrams:** a ray parallel to the axis crosses the axis at F after passing through the lens. A nearer crossing means a smaller  $f$ . Thus if  $f_2 < f_1 < f_3$ , then  $n_2 > n_1 > n_3$ . This preserves the reasoning in the annotated comparison question.

**Conditions matter:** compare the same lens shape, surrounding medium and wavelength. “Larger  $n$  means shorter  $f$ ” is not a rule for changing arbitrary lens shapes or for mirrors.

# Measuring focal length and refractive index

## Experiment A: focal length from several images

Use an illuminated object, convex lens, screen and ruler on a common axis. For several  $s_o > f$  values, move the screen until the image is sharp. Measure  $s_o$  and  $s_i$  from the lens center. Repeat the focusing to estimate uncertainty; do not use object-to-screen distance as  $s_i$ .

### LINEAR GRAPH

Plot  $y = 1/s_i$  against  $x = 1/s_o$ .

$y = -x + 1/f$ : expected slope  $-1$ ; vertical intercept  $1/f$ .

If the fitted intercept is  $5.0 \text{ m}^{-1}$ , then  $f = 0.20 \text{ m}$ .

Choose a range of distances rather than repeating one point. Keep the lens vertical and the object, lens and screen aligned. A large aperture can make blur and aberration more noticeable. Focusing uncertainty becomes severe when the image is very distant.

## Experiment B: refractive index from angles

Trace incident and refracted rays at a flat boundary for several angles. Draw the normal carefully. Measure angles from it, then plot  $\sin \theta_1$  vertically against  $\sin \theta_2$  horizontally. The slope equals  $n_2/n_1$ . With air as medium 1, it is approximately  $n_2$ .

A semicircular block can let one part of the path meet a curved boundary along a radius, so there is no bending there. This isolates the refraction at the flat face. Repeat angle measurements and avoid confusing a surface angle with its complementary normal angle.

## Applying the same lens model to the eye

A normal eye forms a real image on the retina. When an object moves closer, the lens-to-retina distance is almost fixed. In  $1/f = 1/s_o + 1/s_i$ , increasing  $1/s_o$  requires increasing  $1/f$ . The eye accommodates by increasing its converging power, reducing  $f$ .

### PAUSE AND EXPLAIN

Why can a screen locate a real image, but fail to locate a virtual image? Why should an experimental focal-length estimate use many measurements and a fitted graph instead of one potentially uncertain focus?

# Chapter practice: explain and sketch

- 1. Reflection.** A light ray strikes a mirror at  $35^\circ$  to the surface. Find its incidence and reflection angles. Explain why a rough surface can scatter light while each ray obeys the reflection law.
- 2. The road-to-sand model.** Explain why an obliquely incident wave bends toward the normal when it slows. State what happens to frequency and wavelength. Then explain the normal-incidence case.
- 3. Refraction.** Light with vacuum wavelength 500 nm enters glass of  $n = 1.60$  from air at  $40^\circ$ . Find the refracted angle, speed and wavelength in glass.
- 4. Total internal reflection.** Find the water-to-air critical angle for  $n_{\text{water}} = 1.33$ . Describe what happens at incidence  $40^\circ$ , at the critical angle, and at  $55^\circ$ . Could the corresponding air-to-water ray show total internal reflection?
- 5. The converging pair.** A real object is 18 cm from either a convex lens or a concave mirror with  $f = 12$  cm. For each instrument, draw two principal rays and give a complete image description. State exactly which result changes when the instrument changes.
- 6. Work backward.** A converging instrument with  $f = 10$  cm produces an upright image three times the object height. Find  $s_o$  and  $s_i$ . State the physical image side for a lens and for a mirror.

## DISCUSSION BEFORE CALCULATION

For questions 5 and 6, first identify the object-position region and the expected signs of  $s_i$  and  $m$ . After calculating, use the result to check the sketch, not replace it.

## Ray-diagram workspace

# Chapter practice: connect the models

**7. The diverging pair.** A real object is 24 cm from either a concave lens or a convex mirror with  $f = -12$  cm. Find  $s_i$  and  $m$ . Explain the image-side difference and prove that the image lies within one focal-length magnitude of the instrument.

**8. Boundary cases.** For  $f = 8$  cm, describe the image for  $s_o = 16$  cm,  $s_o = 8$  cm, and a very distant object. Explain precisely what “no image at the focus” should mean.

**9. Covering a lens.** A screen shows a sharp image from a convex lens. Cover the upper half of the lens. Predict the image location, size, completeness and brightness, and justify your prediction with rays from one object point.

**10. Material comparison.** Three converging lenses have identical shapes and are surrounded by air. Their focal lengths are  $f_A = 8$  cm,  $f_B = 12$  cm and  $f_C = 20$  cm for the same wavelength. Rank the lens indices and explain the physical reason.

**11. Graph analysis.** A plot of  $1/s_i$  against  $1/s_o$  has slope  $-1.02$  and vertical intercept  $4.0 \text{ m}^{-1}$ . Estimate  $f$ . Is the slope close to the model prediction? Give two practical sources of measurement uncertainty.

**12. Layered tank.** Light enters three horizontal layers with indices  $n_1 < n_2 < n_3$ , all greater than the air index. Derive the bottom-layer angle directly from the entrance angle. After reflection at a horizontal bottom mirror and exit into air, compare the exit and entry angles.

## FINAL ORAL CHECK

A student says: “Positive image distance always means the image is on the right.” Give a counterexample. Another says: “A convex device always converges.” Compare a convex lens and a convex mirror to correct the claim.

# Worked answers: reflection to converging images

## 1-2. Reflection and the analogy

1. The normal is  $90^\circ$  from the surface, so  $\theta_i = \theta_r = 55^\circ$ . A rough surface has varying local normals; the law holds relative to each normal.
2. One part of an oblique wavefront enters the slower medium before the rest and advances less, rotating the wavefront. Its perpendicular ray turns toward the normal. Frequency stays fixed and wavelength decreases. At normal incidence the entire front crosses together: there is no turning, although speed and wavelength change.

## 3-4. Refraction and total internal reflection

3.  $\sin \theta_2 = \sin 40^\circ / 1.60 = 0.4017$ , so  $\theta_2 = 23.7^\circ$ .  
 $v = c/1.60 = 1.875 \times 10^8$  m/s;  $\lambda = 500/1.60 = 312.5$  nm.
4.  $\theta_c = \sin^{-1}(1/1.33) = 48.8^\circ$ . At  $40^\circ$  there is refraction plus partial reflection. At  $\theta_c$  the limiting transmitted ray is along the surface. At  $55^\circ$  there is total internal reflection. Air-to-water light fails the higher-to-lower-index condition.

## 5. One calculation, two physical locations

$$1/s_i = 1/12 - 1/18 = 1/36, \text{ so } s_i = +36 \text{ cm.}$$

$$m = -36/18 = -2.$$

Both images are real, inverted and twice the object height. The lens image is 36 cm beyond the lens; the mirror image is 36 cm in front of the mirror, on the object side. The ray rules change, but the computed image distance and magnification do not.

## 6. Upright and three times as tall

$$m = +3 \text{ implies } s_i = -3s_o.$$

$$1/10 = 1/s_o - 1/(3s_o) = 2/(3s_o).$$

$$s_o = 20/3 \text{ cm} = 6.67 \text{ cm; } s_i = -20 \text{ cm.}$$

The object is inside  $f$ , as predicted. The image is virtual and upright. It is 20 cm from the lens on the object side, or 20 cm behind the concave mirror.

### CHECKING A DRAWING

For question 5, draw the object between  $F$  and  $2F$  and the image beyond  $2F$ . For question 6, the outgoing rays must diverge; only their dashed backward extensions meet at the virtual image.

## Worked answers: comparisons and experiments

7.  $1/s_i = -1/12 - 1/24 = -1/8$ , so  $s_i = -8$  cm and  $m = +1/3$ . Both images are virtual, upright and reduced. The lens image is on the object side; the mirror image is behind the mirror. Here  $|s_i| = 8$  cm  $< |f| = 12$  cm. In general,  $|s_i| = Fs_o/(F+s_o) < F$  for  $F = |f|$ .

8. At  $s_o = 16$  cm  $= 2f$ ,  $s_i = 16$  cm and  $m = -1$ : real, inverted, equal size. At  $s_o = 8$  cm  $= f$ , outgoing rays are parallel and there is no finite image. For a very distant real object, the real image is near  $f$  and is strongly reduced. Apply the lens/mirror side rule in each case.

9. In the ideal model, the image stays at the same position and size, remains complete, and becomes dimmer. Other unblocked rays from each object point can still reach the corresponding image point. Blocking an aperture region is not the same as blocking a region of the object.

10.  $n_A > n_B > n_C$ . With lens shape, surrounding medium and wavelength fixed, stronger refraction corresponds to shorter focal length. The conclusion cannot be generalized to comparisons in which shape also changes.

11.  $f = 1/(4.0 \text{ m}^{-1}) = 0.25$  m. The slope  $-1.02$  is close to the predicted  $-1$ ; whether it agrees statistically depends on experimental uncertainty. Uncertainty in sharpest-screen position and distance measurements are two examples; poor axis alignment can introduce systematic error.

12. At successive parallel boundaries,

$$n_a \sin \theta_i = n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3.$$

Thus  $\theta_3 = \sin^{-1}[(n_a/n_3) \sin \theta_i]$ . Reflection at the horizontal mirror preserves the angle to the normal; the reverse sequence of layers restores the original external angle. The exit angle equals  $\theta_i$ , although the emerging ray is laterally displaced.

### FINAL ORAL CHECK: REASONING

With an object to the left, a concave mirror can have  $s_i > 0$  and a real image on the left. A convex lens in air usually converges; a convex mirror diverges. Decide behavior from the optical device, not the word “convex” alone.

# Sources, coverage and clarifications

The lesson structure and retained examples come from the original handwritten notes below. The paired presentation of convex lens/concave mirror and concave lens/convex mirror follows the teacher's explicit clarification. Repeated drawings are reconciled into labeled constructions, rather than discarded.

| Key | Original source file and pages                  | Used here                                                                                                                  |
|-----|-------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|
| A   | Waves 14.1-14.4 + Optics 13.1-13.3.pdf, pp.4-16 | Reflection; road/sand analogy; rays and wavefronts; Snell; critical angle; fibre; mirrored-tank reasoning                  |
| B   | Waves 14.1-14.6 + Optics 13.3.pdf, p.1          | Snell and total-internal-reflection recap                                                                                  |
| C   | Waves 14.8-14.9 + Optics 13.4.pdf, pp.6-9       | Converging image cases, applications and original lens table                                                               |
| D   | Optics 13.2, 13.4.pdf, pp.1-12                  | Signs; magnification; half-covered lens; diverging systems; numerical pair; mirrors; index comparison; large summary table |

**Syllabus:** supplied *AP Physics 2 Course and Exam Description*, ©2026, Unit 13, PDF pp.101-108 (printed pp.94-101).

Coverage: 13.1 ray model/reflection → pp.2-3,8; 13.2 mirrors → pp.2,9-18,26; 13.3 refraction → pp.3-8; 13.4 lenses → pp.9-20,26. Practice and experimental reasoning appear on pp.20-24. The printed framework's topic order is used as a checklist, not as a replacement lesson order.

**Clarifications:** “no image” at  $f$  is refined to no finite image; “denser” means optically higher-index; all image-case tables specify real objects; the lens/mirror shortcut concerns image-case results, not identical ray paths; normal incidence changes no direction but can change speed and wavelength. Apparent depth is labeled as an extension.

## Independent checks and further reading

OpenStax, *University Physics Volume 3*: [Reflection](#).

OpenStax, *University Physics Volume 3*: [Refraction](#).

OpenStax, *University Physics Volume 3*: [Total internal reflection](#).

OpenStax, *University Physics Volume 3*: [Spherical mirrors](#).

OpenStax, *University Physics Volume 3*: [Thin lenses](#).

The mirrored-tank discussion is a teaching paraphrase of the annotated 2023 AP Physics 2 FRQ 1 already present in source A, not a reproduction of the full exam. The road/sand analogy is redrawn from source A. Clean ray diagrams are original paraxial constructions for this edition.

# Master image summary: learn the pairs together

**Common sign rule:**  $s_i > 0$  real,  $s_i < 0$  virtual;  $m < 0$  inverted,  $m > 0$  upright. “Same/opposite side” is always relative to the object.

| Converging pair: $f > 0$ | Signed image distance | Type            | Orientation / size     | Convex lens: image side | Concave mirror: image side |
|--------------------------|-----------------------|-----------------|------------------------|-------------------------|----------------------------|
| $0 < s_o < f$            | $s_i < 0$             | Virtual         | Upright, enlarged      | Same side               | Opposite side (behind)     |
| $s_o = f$                | At infinity           | No finite image | Outgoing rays parallel | No finite location      | No finite location         |
| $f < s_o < 2f$           | $s_i > 2f$            | Real            | Inverted, enlarged     | Opposite side           | Same side (in front)       |
| $s_o = 2f$               | $s_i = 2f$            | Real            | Inverted, same size    | Opposite side           | Same side (in front)       |
| $s_o > 2f$               | $f < s_i < 2f$        | Real            | Inverted, reduced      | Opposite side           | Same side (in front)       |

| Instrument / focal length            | Object          | Image distance   | Orientation / size / type   | Image side                   |
|--------------------------------------|-----------------|------------------|-----------------------------|------------------------------|
| Concave lens: $f < 0$                | Any real object | $- f  < s_i < 0$ | Upright, reduced, virtual   | Same side as object          |
| Convex mirror: $f < 0$               | Any real object | $- f  < s_i < 0$ | Upright, reduced, virtual   | Opposite side; behind mirror |
| Plane mirror: $f \rightarrow \infty$ | Any real object | $s_i = -s_o$     | Upright, same size, virtual | Opposite side; behind mirror |

**Converging applications:** inside  $f \rightarrow$  magnifying glass / close-up concave mirror; between  $f$  and  $2f \rightarrow$  enlarged real projection; beyond  $2f \rightarrow$  reduced real camera image. At  $2f$  the real image has equal size.

## ONE CALCULATION, THEN TRANSLATE THE SIDE

$$1/f = 1/s_o + 1/s_i; m = h_i/h_o = -s_i/s_o.$$

For the same signed  $f$  and real-object distance, a paired lens and mirror share  $s_i$  and  $m$ . A lens transmits the outgoing rays; a mirror reflects them back. That explains the opposite physical image-side rules.