

Modern Physics

Keep the original narrative: discrete photon packets and matter waves lead into photoelectric evidence, then atomic levels and spectra. Nuclear notation introduces the later blackbody, Compton and nuclear-energy lessons. The source’s packet analogy, graph comparisons and conservation arguments remain explicit.

TEACHING ROUTE

The explanations follow the original lesson sequence. Connections, analogies and worked reasoning remain together. Syllabus additions appear beside the ideas they support.

Lesson group	Focus
Light and matter	Photon packets, de Broglie wavelength and photoelectric effect
Atoms	Standing-wave orbits, allowed energies and spectral lines
Radiation	Photon momentum, blackbody spectra and Compton scattering
Nuclei	Mass defect, reaction bookkeeping, decay probability and half-life

Predict first, draw the situation, then calculate. Use the class questions for discussion and the chapter review for independent practice. Unless stated otherwise, numerical examples use idealized models and the assumptions given on the page.

Source keys N01, N02, and so on refer to the original PDF files listed near the end of this chapter. Source page numbers count PDF pages. CED means the supplied 2026 AP Physics 2 Course and Exam Description.

The original packet analogy: amount per packet and packet count

Fixed frequency: equal packet energy



More photons $\rightarrow$ more total energy

Higher frequency $\rightarrow$ larger energy per photon

Equal-size containers illustrate equal energy packets for one monochromatic frequency. Container size and number of containers are separate variables.

The original sketch contrasts a continuous quantity with repeated one-liter containers. Use that picture to separate energy in one photon from the number of photons. At one fixed frequency, each photon carries hf . Transferring more total energy means transferring more such photons.

RELATIONSHIP AND CONDITIONS

One photon: $E = hf$

Monochromatic beam: total energy = Nhf ; power = $(\Delta N/\Delta t)hf$

The analogy has a boundary: all light does not have one universal packet size. Changing the frequency changes hf . Photons also are not literal bottles with a fixed volume or a hard surface.

PAUSE AND EXPLAIN

Two same-frequency beams have different brightness. Which contains more energetic photons?

Neither: the photons have equal energy. The brighter beam sends more photons per second per unit area.

Photons and the quantum model

RELATIONSHIP AND CONDITIONS

Photon energy: $E = hf = hc/\lambda_0$

Photon momentum in vacuum: $p = E/c = h/\lambda_0$

$h \approx 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$; $1 \text{ eV} \approx 1.60 \times 10^{-19} \text{ J}$; $hc \approx 1240 \text{ eV}\cdot\text{nm}$.

Interference and diffraction reveal wave behavior, while localized energy transfer in photoelectric emission and Compton scattering reveals particle behavior. A photon has zero rest mass and zero electric charge but nonzero energy and momentum. Do not use $p = mv$ with $m = 0$ to conclude that photon momentum is zero.

The frequency of a monochromatic beam fixes the energy per photon. Its power also depends on the number arriving each second: $P = (\text{photon arrival rate})hf$. A more intense beam at fixed frequency has more photons per second per unit area; its individual photons are not more energetic.

Worked example

A 500 nm vacuum-wavelength photon has $E = 1240/500 = \mathbf{2.48 \text{ eV}} = 3.97 \times 10^{-19} \text{ J}$ and $p = h/\lambda = \mathbf{1.33 \times 10^{-27} \text{ kg}\cdot\text{m/s}}$. A 1.0 mW beam at that wavelength carries about $\mathbf{2.52 \times 10^{15} \text{ photons/s}}$.

Use λ_0 for vacuum wavelength in $E = hc/\lambda_0$. When a wave enters a stationary transparent medium, its frequency is unchanged but its wavelength and propagation speed change. The photon energy remains hf for that frequency; inserting the shorter in-medium wavelength into hc/λ without adjusting the speed would give the wrong energy. Electromagnetic momentum inside matter requires care beyond the simple vacuum relation.

Matter waves and de Broglie wavelength

RELATIONSHIP AND CONDITIONS

$$\lambda = h/p$$

For a nonrelativistic particle: $p = mv = \sqrt{2mK}$, so $\lambda = h/\sqrt{2mK}$.

Electrons, atoms and other material particles can diffract and interfere. Their de Broglie wavelength becomes easier to observe when their momentum is small and relevant structures have comparable spacing. A macroscopic object has such a tiny wavelength at everyday momenta that ordinary diffraction is not visible.

Worked example: accelerated electrons

An electron starts from rest and is accelerated through 150 V. $K = 150 \text{ eV} = 2.40 \times 10^{-17} \text{ J}$. Momentum $p = \sqrt{2mK} = 6.61 \times 10^{-24} \text{ kg}\cdot\text{m/s}$. Thus $\lambda = 1.00 \times 10^{-10} \text{ m} = 0.100 \text{ nm}$, comparable to atomic spacings.

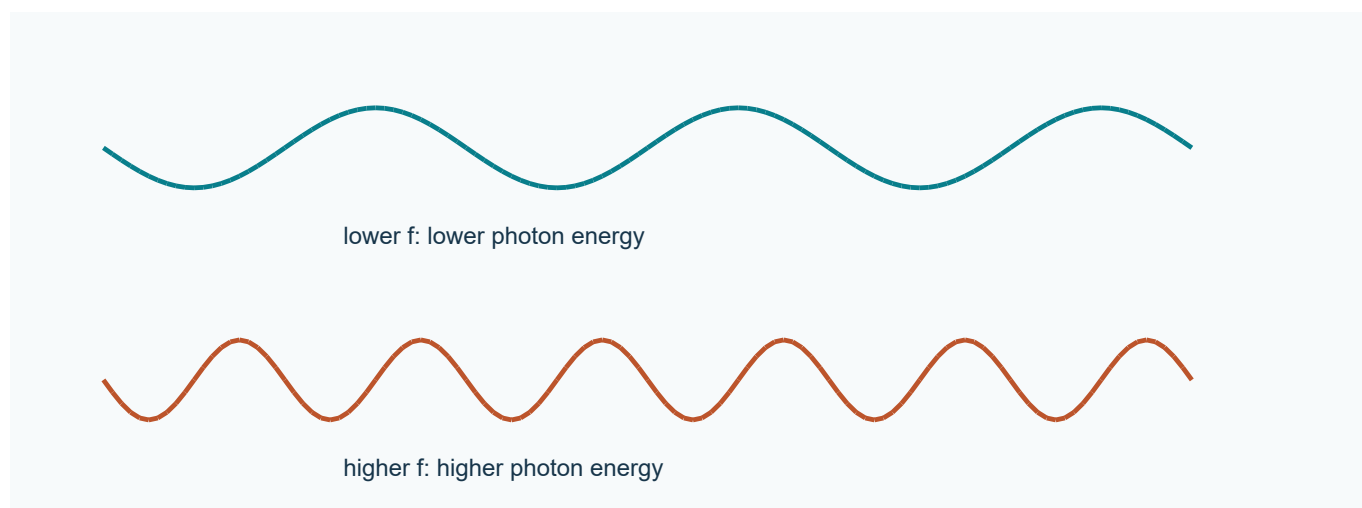
Change	de Broglie wavelength response	Reason
Double speed, same nonrelativistic mass	Halves	p doubles.
Quadruple kinetic energy, same mass	Halves	p scales as $\sqrt{K}$.
Quadruple mass, same kinetic energy	Halves	p scales as $\sqrt{m}$.
Same photon energy versus particle kinetic energy	Do not equate their momenta automatically	Photon $p = E/c$; massive particle p depends on rest mass.

A wave-particle description does not mean the electron physically changes from a tiny ball into an ordinary material string. It means different experiments require different aspects of the quantum description. The Bohr model in the following sections captures useful quantized energy predictions while remaining an approximation.

Check the speed model

The nonrelativistic kinetic-energy relation is appropriate when K is small compared with mc^2 . For an electron, $mc^2 \approx 511 \text{ keV}$. Do not apply $\frac{1}{2}mv^2$ blindly to highly energetic electrons.

The original macroscopic comparison



For light in vacuum, a shorter wavelength means a higher frequency and higher energy per photon. For matter, use its momentum to find wavelength.

A 50 kg person moving at 2.0 m/s has $p = 100 \text{ kg}\cdot\text{m/s}$. The de Broglie wavelength is $\lambda = (6.63 \times 10^{-34})/100 = \mathbf{6.63 \times 10^{-36} \text{ m}}$. This tiny scale explains why ordinary doorway diffraction is not observed for a walking person.

An electron accelerated from rest through a small voltage instead has a wavelength comparable to atomic spacings. For the same nonrelativistic particle, $\lambda \propto 1/\sqrt{V}$. To reduce a 0.100 nm wavelength at 150 V to 0.0500 nm, use four times the voltage: **600 V**.

Comparison	Result
Mass doubles and speed triples	Momentum becomes 6 times larger; λ becomes $\lambda/6$
Kinetic energy doubles, same mass	λ becomes $\lambda/\sqrt{2}$
Photon frequency doubles	Photon energy and momentum double; λ halves

UNITS AND MODEL CHECK

Use joules inside $p = \sqrt{2mK}$ when mass is in kilograms. The nonrelativistic formula is an approximation for massive particles; a photon instead has $E = pc$.

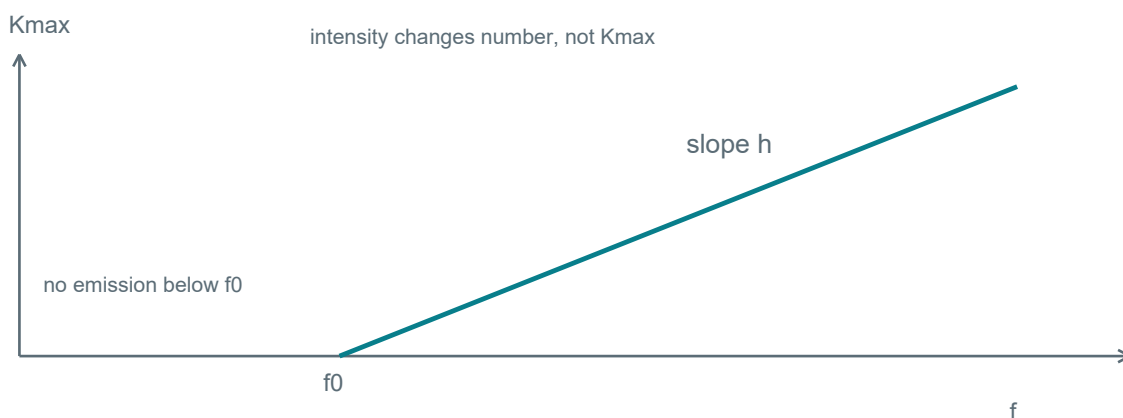
Photoelectric emission

RELATIONSHIP AND CONDITIONS

$$K_{\max} = hf - \phi; f_0 = \phi/h$$

$$\text{Stopping potential magnitude: } eV_{\text{stop}} = K_{\max}$$

A photon can transfer its energy to an electron in a photoactive surface. The work function ϕ is the minimum energy needed to remove an electron from that surface. If hf is below ϕ , no emission occurs in the ordinary single-photon model, however large the intensity. Above threshold, the most energetic emitted electrons have $K_{\max} = hf - \phi$; others can lose energy inside the material.



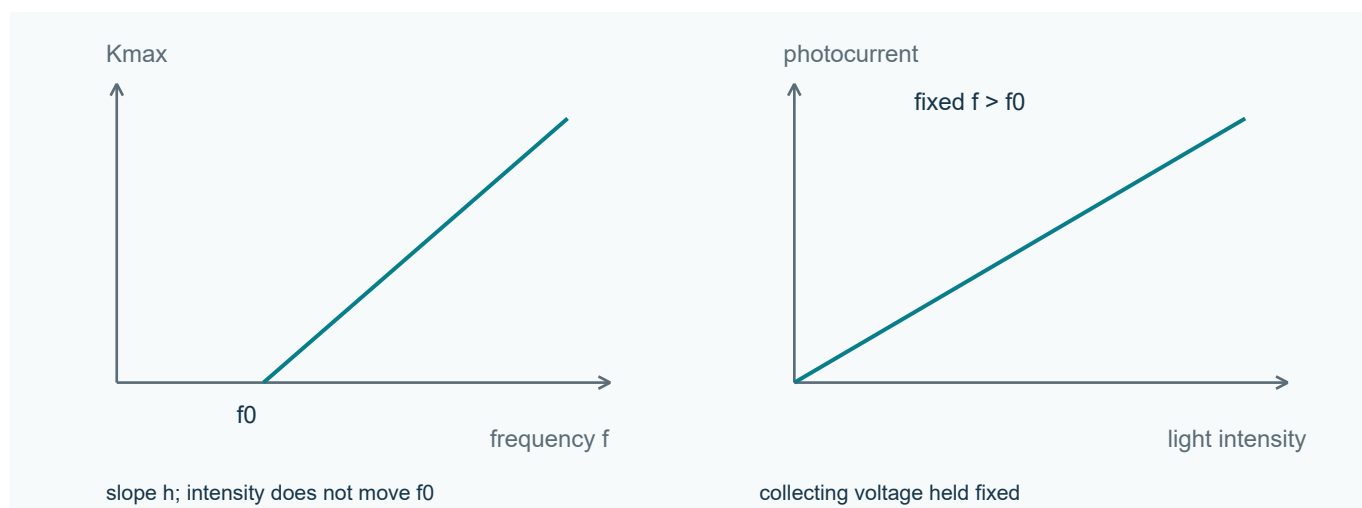
Plotting maximum electron kinetic energy versus light frequency gives slope h , threshold intercept f_0 , and a negative extrapolated vertical intercept $-\phi$. The below-threshold region has no emitted electrons, not negative kinetic energy.

Worked example

A metal has $\phi = 2.0$ eV and receives 400 nm light. Photon energy = $1240/400 = 3.10$ eV, so $K_{\max} = 1.10$ eV and $V_{\text{stop}} = 1.10$ V. At 700 nm, photon energy is 1.77 eV, so ordinary single-photon emission does not occur.

Increasing intensity at fixed above-threshold frequency increases the number of emitted electrons and the saturation current when collection is adequate. It does not increase maximum kinetic energy or stopping potential. Increasing frequency changes the photon energy and therefore changes $K_{\max}$.

The source's photoelectric comparison: energy versus number



Frequency controls the maximum kinetic energy above threshold. Intensity controls electron rate at fixed above-threshold frequency in a model with efficient electron collection.

Change	Maximum electron energy	Electron number / current
Increase intensity at fixed $f > f_0$	Unchanged	Increases with photon arrival rate
Increase intensity at $f < f_0$	No emission in single-photon model	Remains zero
Increase f at fixed beam power	Increases	Not fixed: fewer photons per second may arrive
Use larger work function at same f	Decreases if emission still occurs	May stop if below threshold

The classical gradual-energy-accumulation picture would allow low-frequency light to eject electrons after waiting long enough. The observed threshold and prompt response above threshold instead support transfer in photons. Strong-field multiphoton effects are outside this introductory model.

ORIGINAL QUESTION: DOUBLE THE FREQUENCY

Initially $K_{\text{max}} = hf - \phi$. After doubling f , $K'_{\text{max}} = 2hf - \phi = 2K_{\text{max}} + \phi$. For a positive work function it is greater than twice the old maximum kinetic energy, not merely twice as large.

Photoelectric graphs and an experiment

Illuminate a clean photoactive cathode with several known monochromatic frequencies. For each, adjust the opposing collection voltage until photocurrent just falls to zero. Record the magnitude V_{stop} . Keep the same surface and measurement method, repeat near the cutoff and account for dark current or background illumination.

RELATIONSHIP AND CONDITIONS

$$V_{\text{stop}} = (h/e)f - \phi/e$$

For V_{stop} versus f : slope = h/e , y-intercept = $-\phi/e$.

For K_{max} in joules versus f : slope = h .

Worked data interpretation

Suppose $V_{\text{stop}} = 0.60 \text{ V}$ at $6.0 \times 10^{14} \text{ Hz}$ and 1.43 V at $8.0 \times 10^{14} \text{ Hz}$. Slope $\approx 0.83/(2.0 \times 10^{14}) = 4.15 \times 10^{-15} \text{ V}\cdot\text{s}$. Multiplying by e gives $h \approx 6.64 \times 10^{-34} \text{ J}\cdot\text{s}$. Intercept $\approx -1.89 \text{ V}$, so $\phi \approx 1.89 \text{ eV}$.

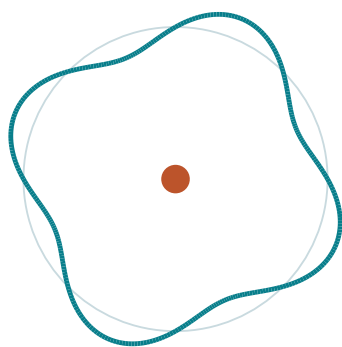
Use a best-fit line through many frequencies rather than relying on two points. Frequency uncertainty, contact potentials, surface contamination and difficulty identifying zero current can bias the result. A larger intensity can improve detectability but must not be mistaken for an increase in stopping potential.

Change in experiment	Photocurrent / number	$K_{\text{max}} / V_{\text{stop}}$
Higher intensity, fixed $f > f_0$	Usually increases if collected	Unchanged in ideal model
Higher f , still above threshold	Depends on photon flux and collection	Increases
Different metal, different ϕ	Threshold and emission can change	Different intercept; same h slope

Atomic structure and standing-wave orbits

An atom has a tiny positively charged nucleus containing protons and neutrons, surrounded by electrons. Most atomic mass is in the nucleus. The proton number Z identifies the element; neutron number distinguishes its isotopes. Changing electron number produces an ion without changing the element. Electron arrangements largely govern chemical interactions.

Example	Protons	Neutrons	Electrons
Carbon-12, neutral	6	6	6
Carbon-14, neutral	6	8	6
Carbon-12 ion with charge +2e	6	6	4



$n = 4$: four wavelengths around the orbit

$$2\pi r = n\lambda$$

$$\lambda = h / p$$

Only matching waves close consistently.

A schematic matter wave with four wavelengths around an allowed circular orbit. The radial wiggles display wave phase; they are not a literal wavy particle trajectory.

RELATIONSHIP AND CONDITIONS

Standing-wave condition: $2\pi r = n\lambda$, where $n = 1, 2, 3, \dots$

With $\lambda = h/(mv)$, this gives $mvr = nh/(2\pi)$.

A wave that fits an integer number of wavelengths rejoins itself with the same phase after one circuit. A noninteger fit does not produce this simple stationary pattern. Combining the condition with Coulomb force supplying centripetal acceleration selects allowed radii and energies rather than a continuous range.

Connect to the next page

The standing-wave condition explains the discrete-state rule in the Bohr model. It does not make the historical orbit picture a complete modern account of an electron; the supplied CED does not require orbital shapes or probability functions.

The Bohr model: allowed states

RELATIONSHIP AND CONDITIONS

Hydrogen: $E_n = -13.6 \text{ eV}/n^2$, $n = 1, 2, 3, \dots$

Circular-orbit model: $ke^2/r^2 = mv^2/r$; $mvr = nh/(2\pi)$

One-electron ion extension: $E_n \approx -13.6Z^2 \text{ eV}/n^2$.



Schematic hydrogen levels (vertical spacing is not to scale) are not evenly separated in energy. The continuum at $E = 0$ corresponds to an electron just unbound with zero kinetic energy in the chosen reference.

The ground state $n = 1$ is the lowest allowed energy; excited states have larger n and less negative energy. Increasing n increases the typical orbital size in the model. The electron cannot have an arbitrary energy between the permitted stationary levels. Negative atomic energy indicates a bound electron relative to a separated electron and proton.

For a circular Coulomb orbit, $K = ke^2/(2r)$ and $U = -ke^2/r$, so total $E = -ke^2/(2r)$. Moving to a larger orbit makes U less negative but reduces K ; total energy still increases because the potential-energy increase is larger than the kinetic-energy decrease. This resolves a common misleading inference that a higher-energy atomic state must always mean a faster electron.

Worked example

Hydrogen at $n = 2$ has $E = -3.40 \text{ eV}$. To ionize from that state with zero final kinetic energy requires **3.40 eV**. From $n = 1$ it requires **13.6 eV**. The energy needed depends on the initial state, not just on the element.

The Bohr model is especially useful for hydrogen-like one-electron systems. Modern quantum mechanics does not treat all electrons as particles following definite classical circular tracks. Use the model where specified rather than as a complete picture of every atom.

Emission and absorption spectra

RELATIONSHIP AND CONDITIONS

Transition photon energy: $hf = |E_{\text{final}} - E_{\text{initial}}|$

Emission: atom moves to a lower level. Absorption: atom gains an allowed energy gap.

A low-density excited gas emits discrete lines because its allowed energy differences are discrete. A cooler gas in front of a continuous source absorbs selected frequencies, producing dark lines in the transmitted spectrum. Different elements have different level structures, so their spectral lines provide identification information.

Worked example: the red hydrogen line

For $n = 3 \rightarrow n = 2$, $E_3 = -1.51 \text{ eV}$ and $E_2 = -3.40 \text{ eV}$. The emitted photon has **1.89 eV**, so $\lambda = 1240/1.89 \approx$ **656 nm**. Reversing this transition requires absorption of the same photon energy by an atom initially in $n = 2$.

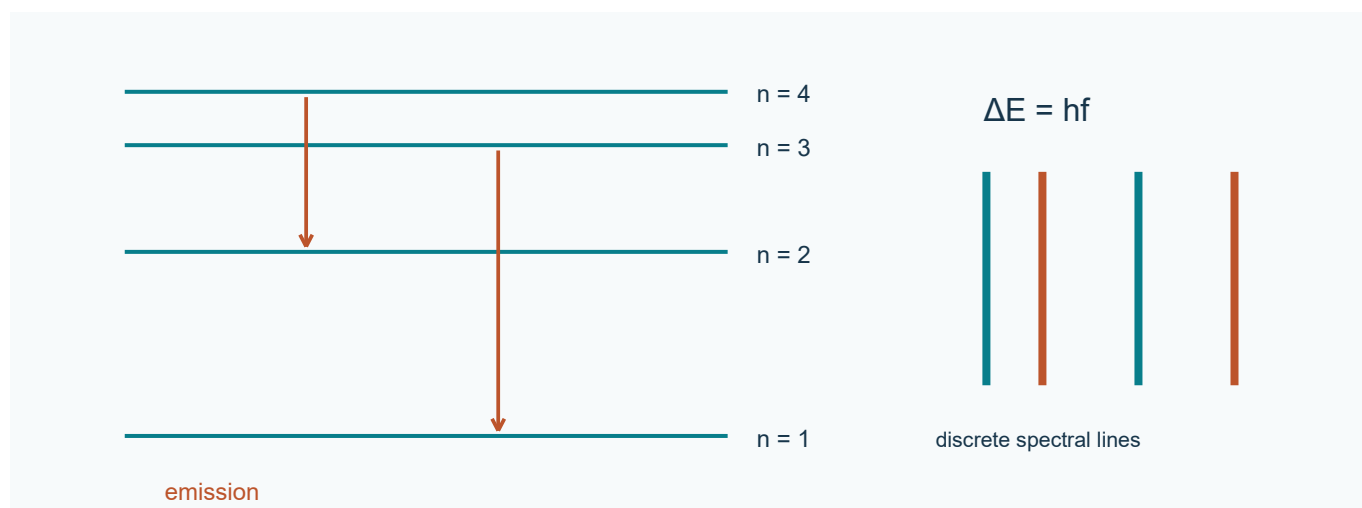
A photon below the ionization threshold can excite an ideal isolated atom only if its energy matches an allowed transition from the occupied initial state, within the line width. A photon above the ionization energy can eject the electron; energy beyond the threshold becomes kinetic energy in the appropriate recoil model. Do not insist that an ionizing photon equal one discrete bound-state gap.

Counting lines carefully

For a sample containing atoms initially at level n that can make all lower transitions, up to $n(n - 1)/2$ distinct lines can appear. For $n = 4$, there are **6 possible lines**. One atom in one cascade does not necessarily emit all six photons; it follows one sequence of transitions.

Larger energy gap means higher frequency and shorter wavelength. The longest-wavelength emitted photon among a set of transitions comes from the smallest nonzero gap, not necessarily the transition involving the lowest principal quantum number.

Six possible lines do not mean six photons from one atom



Each line corresponds to a possible energy gap. Arrows illustrate selected transitions; the level positions here are schematic.

The original $n = 4$ diagram lists transitions $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$. A sample of atoms can populate different cascades and collectively emit all six distinct line energies. One atom can take the direct $4 \rightarrow 1$ route or a cascade such as $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$; it does not follow every route at once.

RELATIONSHIP AND CONDITIONS

Maximum number of distinct pair gaps among n levels: $n(n - 1)/2$

For ideal hydrogen, the shortest wavelength among these transitions comes from $4 \rightarrow 1$, the largest gap. The longest comes from $4 \rightarrow 3$, the smallest gap. For an arbitrary supplied energy-level diagram, compare the stated energy values instead of assuming a hydrogen formula.

ABSORPTION STARTS FROM AN OCCUPIED LEVEL

If every atom is initially in its ground state, the immediately allowed bound-state absorption energies are gaps upward from that state. Do not list every difference between excited levels unless those initial levels are occupied.

Extension retained from the spectrum notes: $1/\lambda = R_H(1/n_{\text{low}}^2 - 1/n_{\text{high}}^2)$. This is the hydrogen transition rule rewritten using wavelength; it is not a separate memorization requirement.

Nuclear notation before nuclear reactions

Symbol / example	Protons Z	Neutrons N	Meaning
A above Z next to X	Z	A - Z	A counts nucleons; Z identifies the element
Helium-4: A = 4, Z = 2	2	2	Neutral atom also has 2 electrons
Uranium-235: A = 235, Z = 92	92	143	A is not a mass value in kilograms
Carbon-12, carbon-13, carbon-14	6 in each	6, 7, 8	Isotopes of one element

Changing neutron number changes isotope. Changing electron number changes ion charge. Changing proton number changes element. Keep these three operations distinct when reading atomic and nuclear questions.

RELATIONSHIP AND CONDITIONS

$A = Z + N$; net atomic charge = $(Z - \text{number of electrons})e$

PAUSE AND EXPLAIN

A carbon-14 ion has charge $+2e$. It still has 6 protons and 8 neutrons, but now has 4 electrons. Its nuclear identity did not change when electrons were removed.

Photon momentum and equal-intensity color comparisons

RELATIONSHIP AND CONDITIONS

Photon: $p = E/c = h/\lambda_0$

At fixed power: photon arrival rate = $P/E = P\lambda_0/(hc)$

The original question compares red and green beams with the same intensity and cross-sectional area. They have equal power. Red photons have lower frequency and therefore less energy each, so the red beam must deliver more photons per second.

Quantity for equal-power red and green beams	Comparison
Photon energy	Red is smaller
Photon momentum magnitude	Red is smaller
Number of photons per second	Red is larger
Speed in vacuum	Equal: c
Total energy crossing per second	Equal: the same power

Original radio-station scale: $P = 50,000 \text{ W}$ and $\lambda = 4.0 \text{ m}$ give $E = hc/\lambda \approx 4.97 \times 10^{-26} \text{ J}$ per photon. The arrival/emission rate is approximately **1.0×10^{30} photons/s**.

PAUSE AND EXPLAIN

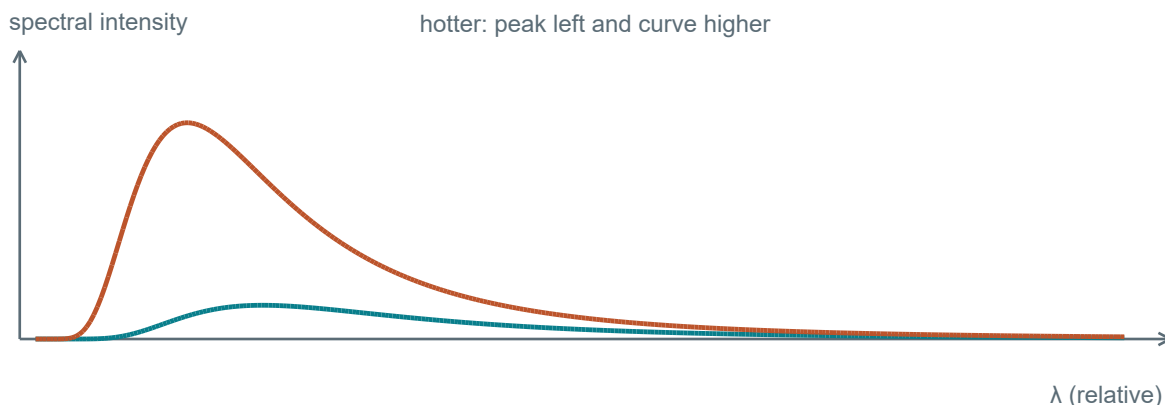
Zero rest mass does not imply zero photon momentum. The relation $p = mv$ is for a nonrelativistic massive particle; the photon relation is $p = E/c$.

Blackbody radiation

RELATIONSHIP AND CONDITIONS

Wien's law: $\lambda_{\text{peak}} T = b \approx 2.90 \times 10^{-3} \text{ m}\cdot\text{K}$

Ideal blackbody emitted power: $P = \sigma AT^4$, $\sigma \approx 5.67 \times 10^{-8} \text{ W}/(\text{m}^2\cdot\text{K}^4)$.



Original Planck-shaped curves at two temperatures. A hotter blackbody has a shorter peak wavelength and greater spectral emission across the wavelength range. The area under each spectral curve gives total emission per area.

A blackbody is an ideal absorber of incident radiation and a temperature-dependent thermal emitter. Its continuous spectrum differs from the line spectrum of an isolated low-density gas. Classical physics could not explain the full distribution; Planck's quantized energy model resolved this failure. The peak-wavelength form of Wien's law refers to a spectrum plotted per unit wavelength.

Worked example

A body with $\lambda_{\text{peak}} = 500 \text{ nm}$ has $T = (2.90 \times 10^{-3})/(5.00 \times 10^{-7}) = \mathbf{5800 \text{ K}}$. Doubling its absolute temperature halves peak wavelength and multiplies emitted power by **16** if area remains fixed.

Extension for real surfaces: emissivity e_m gives $P_{\text{emit}} = e_m \sigma AT^4$. Net radiative loss to large surroundings at T_s is approximately $e_m \sigma A(T^4 - T_s^4)$, under a simple common-emissivity model. Emitted power is not the same as net cooling power when the surroundings also radiate.

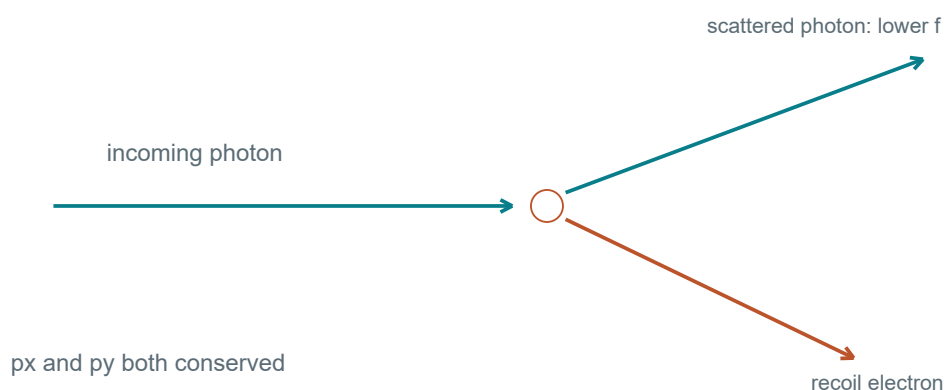
Compton scattering: photons transfer momentum

RELATIONSHIP AND CONDITIONS

For a photon scattering from a free electron initially at rest:

$$\lambda' - \lambda = [h/(m_e c)](1 - \cos \theta)$$

$h/(m_e c) = 2.43 \times 10^{-12} \text{ m}$; θ is the photon's deflection angle.



The incoming photon transfers momentum and energy to the electron. The scattered photon remains a photon; it is not completely absorbed in the Compton process.

At a nonzero scattering angle, the outgoing photon has longer wavelength, lower frequency, lower energy and smaller momentum magnitude. The electron gains kinetic energy. At $\theta = 0$, the ideal Compton wavelength shift is zero; at $\theta = 180^\circ$, the shift is maximal, $2h/(m_e c)$. Both energy and vector momentum are conserved for the full photon-electron system.

Worked wavelength example

An X-ray photon with $\lambda = 0.0500 \text{ nm}$ scatters through 90° . $\Delta\lambda = 0.00243 \text{ nm}$, so $\lambda' = \mathbf{0.05243 \text{ nm}}$. Its energy falls from approximately 24.8 keV to 23.7 keV; the difference, about **1.15 keV**, becomes electron kinetic energy in the initially-rest electron model.

Distinguish the interactions

Photoelectric effect: the incoming photon is absorbed and an electron can leave a bound material.

Compton scattering: a photon emerges with changed momentum after interaction with an approximately free electron. Both support photon energy/momentum, but their equations describe different processes.

Compton momentum in two dimensions

RELATIONSHIP AND CONDITIONS

Incoming photon along +x; outgoing photon at angle θ :

$$p_{\text{ex}} = E/c - (E'/c)\cos \theta$$

$$p_{\text{ey}} = -(E'/c)\sin \theta$$

$$K_{\text{electron}} = E - E'; p_e = \sqrt{(p_{\text{ex}})^2 + (p_{\text{ey}})^2}.$$

Draw the directions first. The electron must balance the outgoing photon's transverse momentum, since the initial y-momentum is zero. Its recoil angle therefore lies on the opposite side of the incoming direction. Conserving only momentum magnitudes misses this essential vector constraint.

Worked example: 100 keV at 90°

An initially stationary electron has rest energy 511 keV. Combining the Compton relation with $E = hc/\lambda$ gives $E' = E/[1 + (E/511 \text{ keV})(1 - \cos \theta)]$. For $E = 100 \text{ keV}$ and $\theta = 90^\circ$, **$E' = 83.6 \text{ keV}$** and **$K_e = 16.4 \text{ keV}$** .

Momentum components are **$p_{\text{ex}} = 100 \text{ keV}/c$** and **$p_{\text{ey}} = -83.6 \text{ keV}/c$** . The electron recoils at angle $-\tan^{-1}(83.6/100) = -39.9^\circ$. Momentum magnitude is **$130 \text{ keV}/c$** .

The notation keV/c is a momentum unit, not a speed. Multiply the numerator by the joules-per-keV conversion and divide by c to express it in kg·m/s. At these energies, electron kinetic energy is not exactly $p^2/(2m)$; use the energy difference $E - E'$ or the relativistic energy-momentum relation if needed. The component momentum equations themselves remain valid.

Direction check

If the scattered photon goes above the original direction, the electron recoils below. At exactly backward photon scattering, both photon momenta are along x, and the recoil electron has no y component.

Fission and fusion: balance before calculating energy

Fission splits a heavy nucleus into smaller nuclei, often with additional neutrons. Fusion combines light nuclei into a heavier nucleus, sometimes with additional particles. Reactions release energy when the products have lower total rest energy, often corresponding to greater binding energy per nucleon. Fission and fusion are not automatically exothermic for every possible nucleus and reaction.

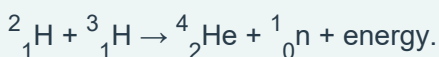
RELATIONSHIP AND CONDITIONS

$$Q_{\text{reaction}} = (\Sigma m_{\text{initial}} - \Sigma m_{\text{final}})c^2$$

$Q > 0$: energy release. $Q < 0$: energy input is required.

Balance nucleon number A and electric charge on both sides.

Illustrative fusion reaction



A : $2 + 3 = 4 + 1$. Charge number: $1 + 1 = 2 + 0$. A neutron carries no electric charge but carries mass, kinetic energy and momentum.

A neutron released by one fission can induce another fission, allowing a chain reaction under suitable conditions. Fusion of positive nuclei requires overcoming electric repulsion sufficiently for the short-range nuclear interaction to act. These are explanations of the energy and force models, not a need to memorize specific isotope reaction chains.

Momentum in a decay at rest

For a parent initially at rest decaying into two products, the products have equal and opposite momenta. If their speeds are nonrelativistic, $K = p^2/(2m)$, so the lighter product receives more kinetic energy. Equal momentum magnitudes do not mean equal speeds or equal kinetic energies.

Worked ratio

An alpha particle and a heavy daughter nucleus recoil from a parent at rest. If the daughter mass is 50 times the alpha mass, the alpha speed is 50 times the daughter speed and its kinetic energy is 50 times as large. Both momentum vectors sum to zero.

Nuclei, mass defect and binding energy

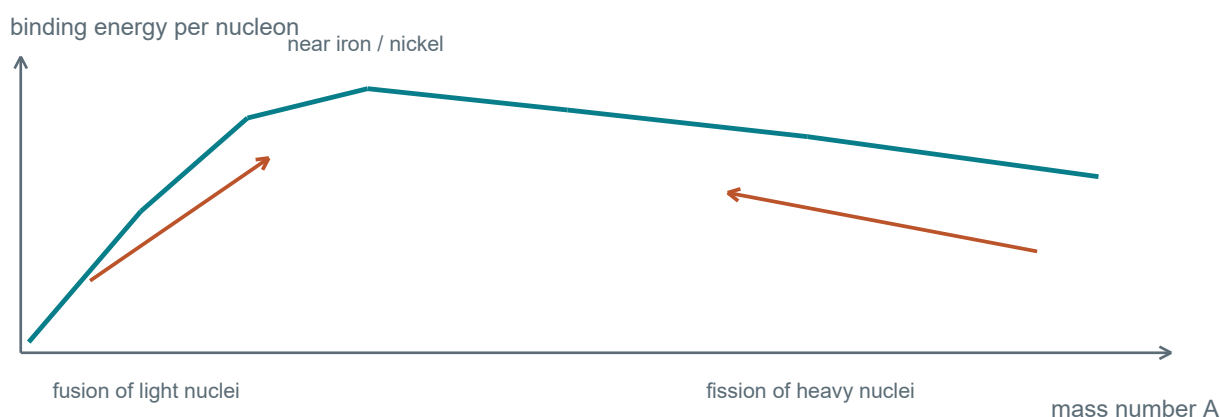
RELATIONSHIP AND CONDITIONS

Nuclear notation: ${}^A_Z\text{X}$; $A = Z + N$

Binding energy: $E_b = \Delta mc^2$

Using nuclear masses: $\Delta m = Zm_p + Nm_n - m_{\text{nucleus}}$

Z is proton number and determines the element. A is nucleon number, not electric charge or a mass measured in kilograms. Isotopes have the same Z and different neutron numbers. The strong nuclear interaction binds nearby nucleons despite electric repulsion among protons; it is important over nuclear length scales.



The qualitative binding-energy-per-nucleon curve rises rapidly for light nuclei, reaches a broad maximum near iron/nickel, then decreases slowly for very heavy nuclei.

A bound nucleus has less rest mass than its separated constituent nucleons. Energy must be supplied to separate them, while energy can be released when a more tightly bound configuration forms. Mass alone is not conserved in a nuclear reaction; total energy and momentum are conserved. Greater binding energy per nucleon generally indicates stronger binding, but comparing total reaction energy still requires the full initial and final masses.

Worked example

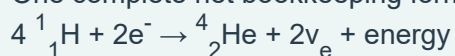
A nuclear reaction has total initial rest mass exceeding final rest mass by 0.0200 u. Using $1 \text{ u} \cdot c^2 \approx 931.5 \text{ MeV}$, the energy release is **18.6 MeV**. This can appear as product kinetic energy and photons, with momentum shared among the products.

Use nuclear masses consistently. If neutral atomic masses are supplied, electron masses often cancel when the number of electrons balances; beta reactions require special care. Do not subtract atomic and nuclear masses indiscriminately.

The original fusion equation and binding-energy argument

RELATIONSHIP AND CONDITIONS

One complete net bookkeeping form:



This is a net accounting equation for converting four protons to a helium nucleus with electrons included on the initial side; it is not a single elementary collision. Nucleon number is 4 on each side. Total charge is $+4e - 2e = +2e$ on the left and $+2e$ on the right. With the source's lepton-number convention, two electrons and two neutrinos each give total lepton number $+2$.

Another common net description writes two positrons among the products instead of two initial electrons. Both representations can be related by including electron-positron annihilation. Always state which particles are included before comparing total masses or released energies.

RELATIONSHIP AND CONDITIONS

For a helium nucleus: $\Delta m = 2m_p + 2m_n - m_{\text{He nucleus}}$

Binding energy = Δmc^2

TWO ENERGY IDEAS TO KEEP APART

Nuclear binding energy is the energy needed to separate a nucleus into nucleons. Electron binding or ionization energy removes an electron from an atom. Both describe bound systems, but their particles and energy scales differ.

For a full reaction, compare total initial and final rest masses. A larger binding energy per nucleon often indicates the favorable direction, but energy release alone does not remove a reaction barrier or determine its rate.

Alpha, beta and gamma decay

Process	Nuclear bookkeeping	Emitted particles
Alpha	A decreases by 4; Z decreases by 2	${}^4_2\text{He}$ nucleus, charge +2e
Beta-minus	A unchanged; Z increases by 1	Electron + antineutrino
Beta-plus	A unchanged; Z decreases by 1	Positron + neutrino
Gamma	A and Z unchanged; nuclear energy falls	High-energy photon

RELATIONSHIP AND CONDITIONS

β^- at nucleon level: $n \rightarrow p + e^- + \text{antineutrino}$

β^+ within an energetically allowed nucleus: $p \rightarrow n + e^+ + \text{neutrino}$

Gamma: $X^* \rightarrow X + \gamma$, where * denotes an excited nuclear state.

A beta electron is created in the nuclear transformation; it is not simply an orbital electron falling out. Neutrinos and antineutrinos carry energy, momentum and lepton number with no electric charge, negligible mass on this scale, and very weak interaction with ordinary matter. Positrons have the electron's mass and opposite charge. An emitted gamma photon reduces nuclear excitation without changing which isotope remains.

Conserve electric charge, nucleon number and lepton number in these reactions. Assign lepton number +1 to electrons and neutrinos, -1 to positrons and antineutrinos. Thus beta-minus emission produces +1 and -1 leptons, totaling zero; beta-plus produces -1 and +1. Energy and momentum conservation also apply.

Worked bookkeeping

${}^{238}_{92}\text{U}$ undergoing alpha decay produces a daughter with $A = 234$ and $Z = 90$. A beta-minus decay of a nucleus with $A = 14$ and $Z = 6$ gives a daughter with **$A = 14$, $Z = 7$** , plus an electron and antineutrino. Specific isotope decay modes need not be memorized for the supplied CED.

Alpha particles are strongly ionizing and have short penetration ranges; beta particles typically penetrate farther, and gamma photons can be much more penetrating. These are qualitative tendencies that depend on energy and material. They do not mean any radiation type is harmless.

Decay bookkeeping: count A and charge separately

alpha

$$\Delta A = -4$$

$$\Delta Z = -2$$

beta minus

$$\Delta A = +0$$

$$\Delta Z = +1$$

beta plus

$$\Delta A = +0$$

$$\Delta Z = -1$$

gamma

$$\Delta A = +0$$

$$\Delta Z = +0$$

Balance nucleon number and charge; include neutrinos in beta decay.

These changes refer to the daughter nucleus. Beta decay keeps nucleon count fixed while changing the neutron-proton balance.

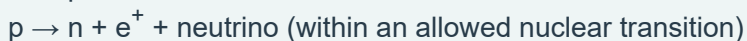
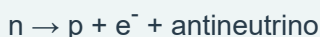
Alpha: write a helium-4 nucleus on the product side, then subtract 4 from A and 2 from Z for the daughter.

Beta minus: a neutron becomes a proton; daughter Z rises by one.

Beta plus: a proton in an energetically allowed nucleus becomes a neutron; daughter Z falls by one.

Gamma: the nucleus loses excitation energy; A and Z stay fixed.

RELATIONSHIP AND CONDITIONS



SOURCE SIGN CORRECTION

In alpha decay the daughter has A - 4 and Z - 2. A plus sign on the daughter's indices would violate conservation. In beta-minus decay the daughter's Z increases, even though the emitted particle has negative charge.

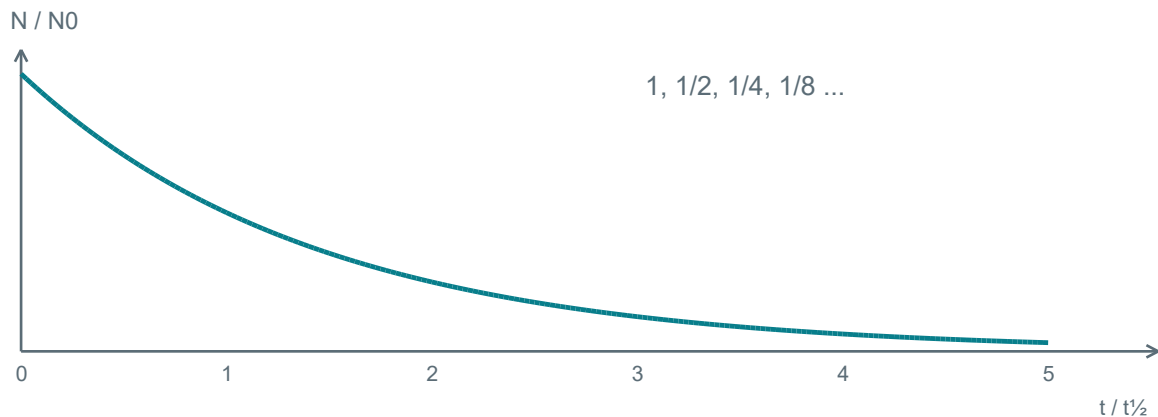
The supplied CED does not require memorizing which isotope has which decay mode or its half-life. Electron capture, standalone neutron-emission decay and detailed neutrino types are outside the stated scope. Keep conservation reasoning with supplied information.

Radioactive decay is statistical

RELATIONSHIP AND CONDITIONS

$$N(t) = N_0 e^{-\lambda t} = N_0 (1/2)^{t/t_{1/2}}$$

$$t_{1/2} = \ln 2 / \lambda; \text{ activity } A_{\text{activity}} = \lambda N \text{ (decays/s = Bq).}$$



The fraction remaining halves during each equal half-life interval. The same fraction, not the same number of nuclei, disappears in each interval.

An individual unstable nucleus has an unpredictable decay time. A large sample has a predictable statistical trend because each surviving nucleus has the same decay probability per unit time in the simple constant-decay-rate model. Half-life is not the time by which every nucleus has decayed; the exponential curve approaches zero asymptotically.

Worked example

A sample initially contains 8.0×10^{12} parent nuclei and has half-life 6.0 h. After 18 h, three half-lives have elapsed, so $N = 1.0 \times 10^{12}$. The number decayed is 7.0×10^{12} . Its activity falls by the same factor of eight if λ stays fixed.

For a stable daughter initially absent in a closed sample with one daughter per parent decay, the daughter count is $N_0 - N$. This relation fails if daughters also decay, products escape or daughter nuclei were initially present. Radioactive dating requires a model for the initial abundance and for whether the system remained closed.

Dating a sample: if the initial amount and decay constant are known, solve $N/N_0 = e^{(-\lambda t)}$ to obtain $t = \ln(N_0/N)/\lambda$. For example, 25% remaining corresponds to two half-lives. Different unstable isotopes have different half-lives, from extremely short intervals to geological times; use supplied values.

Why the half-life stays fixed while activity falls

RELATIONSHIP AND CONDITIONS

Constant probability rate $\lambda \rightarrow dN/dt = -\lambda N$

Solution: $N = N_0 e^{-\lambda t}$

At $t = t_{1/2}$: $1/2 = e^{-\lambda t_{1/2}}$, so $t_{1/2} = \ln 2 / \lambda$

The differential notation retains the original derivation as an optional mathematical explanation. A larger λ means a larger decay probability per unit time and a shorter half-life. λ is a rate with units of inverse time; it is not itself a dimensionless probability over an arbitrary finite interval.

After one half-life, half the original nuclei remain. During the next equal interval, half of those survivors decay, leaving one quarter of the original. The numbers decaying in successive intervals decrease because fewer candidates remain.

Elapsed half-lives	0	1	2	3	4
Fraction remaining	1	1/2	1/4	1/8	1/16
Fraction decayed	0	1/2	3/4	7/8	15/16

PAUSE AND EXPLAIN

Can we predict which particular nucleus will decay next? No. A predictable population trend does not determine each nucleus's time. A long-lived survivor does not use up a scheduled portion of its own lifetime.

The source's decay-and-recoil comparison

A nucleus initially at rest emits an alpha particle and a daughter. With no significant external impulse, total momentum stays zero. In the source's short-decay-timescale idealization, the system's center of mass stays fixed even though each product moves.

RELATIONSHIP AND CONDITIONS

$$p_{\alpha} = -p_{\text{daughter}}$$

$$\text{Nonrelativistic: } K = p^2/(2m)$$

$$K_{\alpha}/K_{\text{daughter}} = m_{\text{daughter}}/m_{\alpha}$$

The lighter alpha particle takes more kinetic energy. Energy conservation uses the rest-mass loss as the source of the total product kinetic energy. If a gamma photon is also emitted, include its momentum and energy; the simple two-body equality need not hold for the two massive particles alone.

Magnetic connection: for equal momentum magnitudes in the same perpendicular B , $r = p/(|q|B)$. A larger charge magnitude then produces the smaller curvature radius. Compare charge as well as mass; equal momentum makes mass cancel from this radius comparison.

GRAPH READING

Find a half-life by reading the time for the parent count to fall from a chosen value to half that value. Use several pairs on a noisy curve. Subtract detector background before treating a count-rate curve as proportional to the number of parent nuclei.

Extension: annihilation and pair production

An electron and positron can annihilate, converting their rest energy and kinetic energy into photons. If the pair is initially at rest as a system, two equal-energy photons emitted in opposite directions can conserve both energy and momentum. One photon alone cannot carry zero net momentum while carrying nonzero energy.

RELATIONSHIP AND CONDITIONS

Electron-positron pair initially at rest:

$$E_{\text{total}} = 2m_e c^2 = 1.022 \text{ MeV}$$

Two-photon annihilation: 0.511 MeV per photon, opposite momenta.

Worked example

Each annihilation photon has wavelength $\lambda = hc/E = (1240 \text{ eV}\cdot\text{nm})/(511000 \text{ eV}) = \mathbf{0.00243 \text{ nm}}$. If the pair initially has kinetic energy, the final photons share the larger total energy and need not have equal laboratory-frame energies.

Pair production is the reverse type of energy conversion: photon energy can create an electron-positron pair when enough energy and a suitable momentum-sharing partner are available. Near a heavy nucleus, the threshold is approximately $2m_e c^2$, with recoil corrections. A single photon in empty space cannot create only a free electron-positron pair while conserving both energy and momentum.

Do not equate rest mass and total mass-energy carelessly

Photons have zero rest mass and still carry energy. The statement $E = mc^2$ refers to rest energy when m is rest mass; total energy for a moving massive particle also includes kinetic energy. Use the full energy budget in every conversion.

Experiments and data interpretation

A | Simulate random decay

Use many identical dice as simulated nuclei. Roll the surviving dice once per time step and remove those showing one selected face. Record survivors after each step; repeat the entire trial and average the counts. Each die has a fixed probability $1/6$ of “decay” per step, so the expected survivor count is $N_0(5/6)^k$. Individual histories vary, especially when few dice remain.

Plot $\ln(N/N_0)$ versus step number for the averaged positive counts. The slope should be $\ln(5/6)$, not $-1/6$ exactly; the dice model has discrete steps. The effective half-life is $\ln(1/2)/\ln(5/6) = 3.80$ steps. This illustrates the difference between a statistical prediction and a schedule for individual events.

B | Analyze measured radiation counts

Measure counts over equal time windows and subtract a separately measured background rate. Plot background-corrected count rate versus time to estimate successive halvings, or use a logarithmic plot to estimate decay constant. Keep detector geometry and settings fixed; changing distance changes collection fraction and can imitate a change in activity.

Directional error

If a constant background is not subtracted, the measured count rate approaches that background instead of zero. A late-time logarithmic fit then appears too shallow and can overestimate half-life. Longer count windows reduce random relative uncertainty but blur rapid changes.

For spectral analysis, measure line positions with a calibrated grating, convert angles to wavelengths and then to energy gaps. Compare predicted gaps, not just color names. Instrument calibration and overlapping spectral orders are important limitations.

Student practice

1. Find the energy in eV and momentum of a 620 nm vacuum-wavelength photon.
2. A particle's nonrelativistic kinetic energy quadruples. What happens to its de Broglie wavelength?
3. Hydrogen transitions from $n = 4$ to $n = 2$. Find emitted photon energy and wavelength.
4. How much energy ionizes hydrogen from $n = 3$?
5. A blackbody temperature rises from 300 K to 600 K at fixed area. Give the peak-wavelength and emitted-power ratios.
6. A metal has work function 2.2 eV and receives 400 nm light. Find K_{max} and stopping potential.

Review: radiation and nuclear processes

7. A photon scatters from a stationary free electron through 180° . Find the maximum wavelength increase.
8. A sample has half-life 5.0 days. What fraction remains after 15 days, and what fraction has decayed?
9. Find λ for half-life 10 s. If $N = 1000$, find activity.
10. An isotope with $A = 210$ and $Z = 84$ emits an alpha particle. Give daughter A and Z .
11. An isotope undergoes beta-minus decay. What happens to A and Z , and what other particles are emitted?
12. A reaction loses 0.0050 u of total rest mass. Find released energy in MeV.

Practice solutions

1. $E = 1240/620 = \mathbf{2.00 \text{ eV}}$; $p = h/\lambda = \mathbf{1.07 \times 10^{-27} \text{ kg}\cdot\text{m/s}}$.
2. $\lambda \propto 1/\sqrt{K}$, so wavelength **halves**.
3. $E_4 = -0.850 \text{ eV}$ and $E_2 = -3.40 \text{ eV}$. Photon energy = **2.55 eV**; $\lambda \approx \mathbf{486 \text{ nm}}$.
4. From $E_3 = -13.6/9 \text{ eV}$ to zero: **1.51 eV**.
5. $\lambda_{\text{new}}/\lambda_{\text{old}} = \mathbf{1/2}$; $P_{\text{new}}/P_{\text{old}} = \mathbf{16}$.
6. Photon energy = 3.10 eV. $K_{\text{max}} = \mathbf{0.90 \text{ eV}}$; stopping potential = **0.90 V**.

Answers: radiation and nuclear processes

7. $\Delta\lambda = 2h/(m_e c) = 4.86 \times 10^{-12} \text{ m}$.

8. Three half-lives: **1/8 remains, 7/8 has decayed**.

9. $\lambda = \ln 2/10 = 0.0693 \text{ s}^{-1}$; activity $\lambda N = 69.3 \text{ Bq}$.

10. Daughter **A = 206, Z = 82**.

11. A stays fixed, Z increases by 1. An **electron and antineutrino** are emitted.

12. $Q = 0.0050(931.5) = 4.66 \text{ MeV}$.

Original notes and syllabus map

The source sequence determines the teaching order. Repeated examples are consolidated while distinct reasoning, diagrams and comparisons are retained.

Original source	Content retained
N23, pp.1-6 Modern Physics 15.1-15.3, 15.5.pdf	Packet analogy → matter wavelength → photoelectric setup and graphs
N24, pp.1-11 Modern Physics 15.1-15.3, 15.5, 15.7-15.8.pdf	Bohr standing waves, spectra, notation, photon momentum and comparisons
N25, pp.1-17 Modern Physics 15.1-15.8.pdf	Blackbody, Compton, fission/fusion, mass defect, decay and extensions
N27, pp.5-6 Problem Solving - Electrostatics 10.1-10.5, 10.7 + Magnetism 12.2 + Modern Physics 15.7-15.8.pdf	Half-life and recoil/center-of-mass reasoning
N30, pp.8-11 Problem Solving - Electrostatics 10 + Circuits 11 + Magnetism 12 + Modern Physics 15.pdf	Modern-physics review questions

Curriculum: supplied *AP Physics 2 Course and Exam Description*, 2026, Unit 15. Topic labels on each content page show where the syllabus connects to the original lesson. Extensions remain labeled.

Clarifications and independent references

Point checked	Treatment in this chapter
Photon packets	Frequency sets packet energy; intensity and power also depend on photon count.
Photoelectric graphs	Slope is h for K in joules versus f ; h/e for stopping voltage versus f .
Line counting	Six possible lines from four levels describe an ensemble, not six photons from every atom.
Alpha daughter	A decreases by 4 and Z by 2.
Decay rate	Larger λ gives shorter half-life. λ is a rate, not a finite-interval probability.
CED additions	Two-dimensional Compton momentum is core; annihilation and pair production are retained extensions.

Independent checks

[OpenStax: photoelectric effect](#)

[OpenStax: Compton effect](#)

[OpenStax: nuclear reactions](#)

Constants and calculations use the values stated in each example. Historical practice provides context and reasoning patterns. New worked questions and corrected explanations are distinguished from an official answer key.

Master summary: choose the correct energy model

Phenomenon	Relationship		Interpretation / main check
Photon	$E = hf = hc/\lambda_0$; $p = E/c$		Vacuum wavelength; zero rest mass does not mean zero momentum
Matter wave	$\lambda = h/p$; $\lambda = h/\sqrt{(2mK)}$		Second form is nonrelativistic
Photoelectric	$K_{\max} = hf - \phi$; $eV_{\text{stop}} = K_{\max}$		f controls maximum energy; intensity controls photon rate
Atomic transition	$E_{\text{photon}} = E_f - E_i $		Emission downward; absorption upward from occupied state
Hydrogen levels	$E_n = -13.6 \text{ eV}/n^2$		Negative means bound; ionization reaches $E = 0$
Blackbody	$\lambda_{\text{peak}}T = b$; $P = \sigma AT^4$		Absolute T ; continuous spectrum
Compton	$\Delta\lambda = h(1 - \cos \theta)/(m_e c)$		Initially stationary free electron; conserve vector momentum
Nuclear energy	$Q = (\sum m_i - \sum m_f)c^2$		Consistent mass convention; include all products
Radioactive decay	$N = N_0 e^{(-\lambda t)}$; $t_{1/2} = \ln 2/\lambda$		Same fractional loss per interval; individual events random
Decay	Daughter A	Daughter Z	Other emitted particles
Alpha	$A - 4$	$Z - 2$	Helium-4 nucleus
Beta minus / beta plus	A unchanged	$Z + 1 / Z - 1$	Electron + antineutrino / positron + neutrino
Gamma	A unchanged	Z unchanged	Photon; nuclear excitation decreases